LOS BASED ATTITUDE ALIGNMENT OF TWO SPACECRAFT IN FORMATION

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Abstract

In this paper we propose a novel control law for achieving attitude alignment and formation keeping of a two spacecraft formation making use of line of sight (LOS) measurements in each spacecraft's own body frame. The proposed control law achieves formation keeping, velocity synchronization, and alignment of attitudes of spacecraft about LOS asymptotically.

Keywords: LOS based control law, Formation Keeping, Attitude Alignment.

1 Introduction

Formation flying is enabling technology which is eagerly sought for future space missions. Many of the planned formation flying missions require precise control of positions as well as orientations of spacecraft used in the mission. Thus combined attitude and position control is a necessary part of formation flying design. Here we consider coupled attitude and position control of a two spacecraft formation where attitudes of spacecraft are not measured and only angular velocities, LOS unit vectors and relative velocity measurements are known. The objective is to achieve a formation keeping with desired distance between two spacecraft and alignment of attitudes along inertial LOS unit vector.

Authors Scharf et al. in their survey papers [1] and [2] on formation flying divides the literature on the area in to two categories based on dynamic environment: Deep Space (DS) where relative spacecraft dynamics reduce to double integrator form and Planetary Orbital Environments (POE), where spacecraft are subjected to significant orbital dynamics and environmental disturbances. We here consider Deep Space like environment with double integrator like relative translational dynamics.

There are many works in literature that make use of LOS unit vector measurements for relative navigation or relative orbit determination of spacecraft. The idea of inertial LOS based cyclic formation control is proposed in [4]. Relative orbit determination using LOS vectors is considered in [3], to cite a few. The idea that LOS unit vectors in respective body frames of individual spacecraft can be used for relative attitude determination is more recent. [5] shows that deterministic relative attitude determination is possible for a formation of three vehicles. Lee in [7] proposes a control law to asymptotically stabilize relative attitude between two spacecraft making use of line of sight (LOS) direction observations between two spacecraft, and LOS direction observations to a common object. But in [7] the relative positions of two spacecraft and the common object are assumed to be fixed. We consider the combined problem of attitude alignment and formation keeping of two spacecraft and hence the relative positions of spacecraft are not fixed. Unlike [7] no common object is considered. Here the problem of simultaneously achieving attitude alignment along the LOS vector and formation keeping making use of only relative measurements in each spacecraft's own body frame is considered. Attitudes of two spacecraft get aligned along LOS such that LOS vector between two spacecraft is represented in same way in body frames of both spacecraft. No absolute measurement is needed for the control law. All the measurements and control inputs are in local coordinates frame. Final formation achieves velocity synchronization.

This paper is organized as follows. The problem is formulated in section 2. Section 3 gives the error functions used, the proposed control law and proof for asymptotic convergence. Section 4 illustrates by a numerical example. A summary is given in section 5.

2 Problem Formulation

2 Dynamics

The attitude of a spacecraft is the orientation of its body fixed frame with respect to the inertial reference frame. This is represented by a rotational matrix in special orthogonal group given as SO(3) $= \left\{R|\ R^T R = I,\ \det(R) = 1\right\}$. Equations of motion of the attitudes of i^{th} spacecraft, for $i = 1, 2$ is given by

$$\begin{aligned} \dot{R}_i &= R_i \mathsf{S}(\Omega_i) && (1)\\ J_i \dot{\Omega}_i &= J_i \Omega_i \times \Omega_i + \tau_i && (2) \end{aligned}$$

where $J_i \in \mathbb{R}^{3\times 3}$ is the moment of inertia, $\Omega_i \in \mathbb{R}^3$ the angular velocity, and $\tau_i \in \mathbb{R}^3$ the control torque in of i^{th} spacecraft in its own body fixed frame. Here S(.) is the map given by S(.) : $\mathbb{R}^3 \to so(3)$ such that $\mathsf{S}(x)y = x \times y$ for any $x, y \in \mathbb{R}^3$. Translational dynamics of the centre of mass of spacecraft is assumed to be double integrator dynamics. Translational equations of motion of the i^{th} spacecraft is given by

$$\begin{aligned} m_i \dot{r}_i &= m_i v_i && (3)\\ m_i \dot{v}_i &= f_i && (4) \end{aligned}$$

where $m_i \in \mathbb{R},\ m_i > 0$, is the mass of the spacecraft and $r_i, v_i \in \mathbb{R}^3$ are position and velocity of i^{th} spacecraft in inertial frame. $f_i \in \mathbb{R}^3$ is force applied on i^{th} spacecraft represented in the inertial frame. We take

$$u_i = \frac{f_i}{m_i} \tag{5}$$

$u_i \in \mathbb{R}^3$ represents the translation control input for i^{th} spacecraft in inertial frame

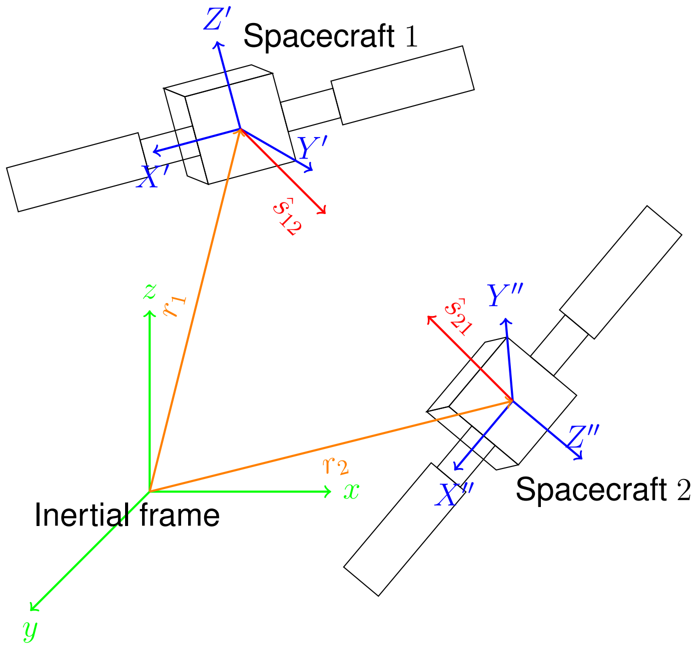


Figure 1: Two spacecraft formation : Here x, y, z is the inertial reference frame. Position vector of centre of mass of spacecraft 1. The body fixed frames of spacecraft 1 is given by X', Y', Z'. Similarly position vector of centre of mass of spacecraft 2 is r_2 and body fixed frame of spacecraft 2 is given by X'', Y'', Z''. Inertial LOS unit vectors $\hat{s}_{12}$ and $\hat{s}_{21}$ are also illustrated in the figure.

2 Measurement strategies

Let $(i, j) \in \{(1, 2), (2, 1)\}$. Let $\widehat{s}_{1j}$ denote the line of sight unit vector observed from the i^{th} spacecraft to the j^{th} spacecraft, represented in the inertial frame. This is given by

$$\widehat{s}_{ij} = \frac{(r_j - r_i)}{\|(r_j - r_i)\|} \tag{6}$$

But measurements are made in each spacecraft's own body frame. We define $\widehat{b}_{1j}$ as the line of sight unit vector observed from the i^{th} spacecraft to the j^{th} spacecraft, represented in the i^{th} body fixed frame. This is given by

$$\widehat{b}_{ij} = R_i^{\top}(\hat{s}_{ij}) = R_i^{\top}\frac{(r_j - r_i)}{\|(r_j - r_i)\|} \tag{7}$$

This is illustrated by figure 1. In addition to LOS unit vectors each spacecraft measures relative velocity with respect to each other. We define $\mathbf{v_{ij}}$ as relative velocity of j^{th} spacecraft observed from the i^{th} spacecraft represented in the i^{th} body fixed frame, given by

$$\mathbf{v_{ij}} = R_i^{\top}(v_j - v_i) \tag{8}$$

Also let the distance between spacecraft be denoted by d. The spacecraft i has access to relative LOS unit vectors $\widehat{b}_{ij}$, relative velocity $\mathbf{v_{ij}}$, distance between the spacecraft d, and own angular velocity Ω_i and $\widehat{b}_{ji}$ which is communicated from spacecraft j. The derivatives of these quantities expressed in terms of $\mathbf{v}_{ij},\ \hat{b}_{ij},\ d$ will be helpful for analysis. Time derivative of d, when $d \neq 0$ is given by

$$\frac{d}{dt}(d) = \frac{1}{2}(\hat{b}_{12} \cdot \mathbf{v}_{12} + \hat{b}_{21} \cdot \mathbf{v}_{21}) \tag{9}$$

Note that $\forall\, u, v \in \mathbb{R}^3$, and $R \in$ SO(3),

$$u \cdot v = (Ru) \cdot (Rv) \tag{10}$$

By making use of (10), the time derivative of LOS direction vector $\hat{b}_{ij}, \{i,j\} \in \{(1,2),(2,1)\}$

$$\frac{d}{dt}(\hat{b}_{ij}) = \hat{b}_{ij} \times \Omega_i + \frac{1}{d}\mathbf{v}_{ij} - \frac{1}{2}(\hat{b}_{12} \cdot \mathbf{v}_{12} + \hat{b}_{21} \cdot \mathbf{v}_{21})\hat{b}_{ij} \tag{11}$$

We make use of following assumption are used in the paper.

(A1) Two LOS unit vectors namely $\widehat{b}_{12}$ and $\widehat{b}_{21}$, and distance between spacecraft d are available to each spacecraft. In addition, each spacecraft i has access to the relative velocity vector in own body frame $\mathbf{v_{ij}}$, and own angular velocity Ω_i.

(A2) The initial inertial positions of the two spacecraft do not coincide, i.e. $r_1(0) \neq r_2(0)$. Mathematically we cannot define LOS unit vector if $r_1 = r_2$. If $r_1(0) \neq r_2(0)$ our control law ensures that $r_1(t) \neq r_2(t)$ for all $t > 0$.

2 Control Objective

We consider two spacecraft having dynamics given in (2)-(3) and which are able to communicate LOS unit vectors with each other. Now the control objective is to achieve,

1. Formation keeping: in other words keeping a desired distance d_0 between the two spacecraft: $\lim\limits_{t\to\infty} \|(r_1(t) - r_2(t))\| = d_0$,
2. Velocity synchronization: $\lim\limits_{t\to\infty} \|v_1(t) - v_2(t)\| = 0$,
3. Attitude alignment along LOS vector: $\lim\limits_{t\to\infty} \|\widehat{b}_{12}(t) + \widehat{b}_{21}(t)\| = 0$ and $\lim\limits_{t\to\infty} \Omega_1(t) = 0$, $\lim\limits_{t\to\infty} \Omega_2(t) = 0$

Here the first objective is to maintain constant distance between the two spacecraft. Second objective is to maintain formation by keeping the inertial LOS vector constant. Third control objective attempts to align attitudes of the two spacecraft along the inertial LOS, such that LOS unit vector is same in both spacecraft's body frames. Thus it is aligning payloads of the two spacecraft point at the same direction along the inertial LOS direction.

3 Combined Attitude and Position Control

Each of the control objectives given above can be represented as minimization of a positive semi definite error function. We make use of three such error functions alignment error function, distance error function and velocity synchronization error function.

Alignment Error Function

We make use of an alignment error function of the form

$$\Psi_1 = \frac{1}{2}\left\|\widehat{b}_{12} + \widehat{b}_{21}\right\|^2 \tag{12}$$

This error function is similar to that used in [6] and [7]. Now from elementary calculation we have

$$\begin{aligned}\Psi_1 &= \frac{1}{2}\left\|\widehat{b}_{12} + \widehat{b}_{21}\right\|^2 = \frac{1}{2}\left[\widehat{b}_{12}\cdot\widehat{b}_{12} + \widehat{b}_{12}\cdot\widehat{b}_{21} + \widehat{b}_{21}\cdot\widehat{b}_{21} + \widehat{b}_{21}\cdot\widehat{b}_{12}\right] \\ &= \frac{1}{2}\left[2 + 2\widehat{b}_{12}\cdot\widehat{b}_{21}\right] = \left[1 + \widehat{b}_{12}\cdot\widehat{b}_{21}\right]\end{aligned}$$

Clearly $\Psi_1 \geq 0$ and $0 \leq \Psi_1 \leq 2$. For $\widehat{b}_{12} = -\widehat{b}_{21}$ we have $\Psi_1 = 0$ and for $\widehat{b}_{12} = \widehat{b}_{21}$ we have $\Psi_1 = 2$. The time derivative of the error function is obtained to be

$$\dot{\Psi}_1 = (\hat{b}_{21} \times \hat{b}_{12})\cdot\Omega_1 + (\hat{b}_{12} \times \hat{b}_{21})\cdot\Omega_2 + \frac{1}{d}\hat{b}_{21}\cdot\mathbf{v_{12}} + \frac{1}{d}\hat{b}_{12}\cdot\mathbf{v_{21}} - \frac{1}{d}(\hat{b}_{12}\cdot\hat{b}_{21})\left(\hat{b}_{12}\cdot\mathbf{v}_{21} + \hat{b}_{21}\cdot\mathbf{v}_{21}\right) \tag{13}$$

The time derivative of error function has additional terms compared to [6] as we consider relative translational dynamics of spacecraft too. The detailed calculation is given in the Appendix.

Distance Error Function

We define a distance error function Ψ_2 of the form

$$\Psi_2(r_1, r_2) = \left(\|r_1 - r_2\| - d_0\right)^2 = (d - d_0)^2 \tag{14}$$

The distance error function is positive semi definite and $\Psi_2 = 0$ only when $\|r_1 - r_2\| = d_0$. The derivative of Ψ_2 is given by

$$\dot{\Psi}_2 = 2\big(d - d_0\big)\dot{d} = \big(d - d_0\big)(\hat{b}_{12}\cdot\mathbf{v}_{12} + \hat{b}_{21}\cdot\mathbf{v}_{21})$$

Velocity Synchronization Error Function

Similar to above velocity synchronization can be considered as a minimization of an error function. We define a velocity synchronization error function Ψ_3 of the form

$$\Psi_3(v_1, v_2) = \frac{1}{2}\|v_1 - v_2\|^2 \tag{15}$$

The velocity synchronization error function is positive semi definite and $\Psi_3 = 0$ only when $\|v_1 - v_2\| = 0$. Additionally the time derivative is given by

$$\dot{\Psi}_3 = (v_1 - v_2)\cdot(u_1 - u_2)$$

From (10) we can write

$$(v_1 - v_2) \cdot u_1 = -R_1^\top (v_2 - v_1) \cdot (R_1^\top u_1) = -\mathbf{v_{12}} \cdot (R_1^\top u_1)$$
$$(v_2 - v_1) \cdot u_2 = -R_2^\top (v_1 - v_2) \cdot (R_2^\top u_2) = -\mathbf{v_{21}} \cdot (R_2^\top u_2)$$

and thus

$$\dot{\Psi}_3 = -\mathbf{v_{12}} \cdot (R_1^\top u_1) - \mathbf{v_{21}} \cdot (R_2^\top u_2) \tag{16}$$

A linear combination of error functions Ψ_1, Ψ_2, Ψ_3 and rotational kinetic energy of the two spacecraft is used as a Lyapunov-like function to prove stability results.

3 Control Law

We now propose a combined attitude and position control law as follows:

$$\tau_1 = -k_{\Omega_1}\Omega_1 - k_1(\widehat{b}_{21} \times \widehat{b}_{12}) \tag{17}$$
$$\tau_2 = -k_{\Omega_2}\Omega_2 - k_1(\widehat{b}_{12} \times \widehat{b}_{21}) \tag{18}$$
$$R_1^\top u_1 = k_{v_1}(\mathbf{v_{12}}) - k_2\big(d_0 - d\big)\widehat{b}_{12} - \tfrac{k_1}{d}\left(\widehat{b}_{21} - \big(\widehat{b}_{12} \cdot \widehat{b}_{21}\big)\widehat{b}_{12}\right) \tag{19}$$
$$R_2^\top u_2 = k_{v_2}(\mathbf{v_{21}}) - k_2\big(d_0 - d\big)\widehat{b}_{21} - \tfrac{k_1}{d}\left(\widehat{b}_{12} - \big(\widehat{b}_{12} \cdot \widehat{b}_{21}\big)\widehat{b}_{21}\right) \tag{20}$$

where control gains $k_{\Omega_1}, k_{\Omega_2}, k_{v_1}, k_{v_2}, k_1, k_2 > 0$. In the torque equations (17)(18) the first terms correspond to dissipation, and the remaining term is chosen to minimize the error function Ψ_1. In position control laws (19),(20) terms are chosen to minimize error functions Ψ_3, Ψ_2 and Ψ_1 respectively. Since we assume that the two spacecraft have no access to inertial coordinates, the control needs to be expressed in the local frame. Position control input u_i expressed in i^{th} spacecraft's body frame is $R_i^\top u_i$.

Proposition 3.1. *Consider the system of equations under the control laws given in (17)-(20) and let the assumptions ($A1$)-($A2$) be satisfied. Then following results hold*

(i) There exist two equilibrium configurations given by

$$Q_1 = \{(R_1, \Omega_1, R_2, \Omega_2, r_1, v_1, r_2, v_2) | \Omega_1 = \Omega_2 = 0, v_1 = v_2,\ \|r_1 - r_2\| = d_0,\ R_1^\top (r_1 - r_2) = R_2^\top (r_1 - r_2)\} \tag{21}$$
$$Q_2 = \{(R_1, \Omega_1, R_2, \Omega_2, r_1, v_1, r_2, v_2) | \Omega_1 = \Omega_2 = 0, v_1 = v_2,\ \|r_1 - r_2\| = d_0,\ R_1^\top (r_1 - r_2) = -R_2^\top (r_1 - r_2)\} \tag{22}$$

(ii) Undesired equilibrium configuration Q_2 is unstable.

(iii) Desired equilibrium configuration Q_1 is asymptotically convergent. A conservative region of attraction is given by

$$\Psi_1(0) < 2 \tag{23}$$
$$\sum_{i=1}^{2} \lambda_{max}(J_i)\|\Omega_i(0)\|^2 + \Psi_2(0) + k_2(\|v_1(0) - v_2(0)\|^2) < 2k_1 - \Psi_1(0) \tag{24}$$

where $\lambda_{max}(J_i)$ is the largest eigen value of J_i.

Proof. : We make use of the LaSalle invariance principle and Chetaev's instability theorem for the proof.

(i) Consider a Lyapunov like function

$$\mathcal{V} = k_1\Psi_1 + k_2\Psi_2 + \Psi_3 + \frac{1}{2}\Omega_1 \cdot (J_1\Omega_1) + \frac{1}{2}\Omega_2 \cdot (J_2\Omega_2) \tag{25}$$

Clearly $\mathcal{V} \geq 0$, and $\mathcal{V} = 0$ only when $v_1 = v_2$, $\|r_1 - r_2\| = d_0$, $\Omega_1 = \Omega_2 = 0$ and $\widehat{b}_{12} = -\widehat{b}_{21}$, i.e. $(R_1, \Omega_1, R_2, \Omega_2, r_1, v_1, r_2, v_2) \in Q_1$. Now taking the time derivative we have

$$\begin{aligned}
\dot{\mathcal{V}} = k_1 & \left[(\hat{b}_{21} \times \hat{b}_{12}) \cdot \Omega_1 + (\hat{b}_{12} \times \hat{b}_{21}) \cdot \Omega_2 + \frac{1}{d}\hat{b}_{21} \cdot \mathbf{v_{12}} + \frac{1}{d}\hat{b}_{12} \cdot \mathbf{v_{21}}\right] \\
& - \frac{k_1}{d}(\hat{b}_{12} \cdot \hat{b}_{21})\left(\hat{b}_{12} \cdot \mathbf{v}_{21} + \hat{b}_{21} \cdot \mathbf{v}_{21}\right) + k_2\big(d - d_0\big)(\hat{b}_{12} \cdot \mathbf{v}_{12} + \hat{b}_{21} \cdot \mathbf{v}_{21}) \\
& - \mathbf{v_{12}} \cdot (R_1^\top u_1) - \mathbf{v_{21}} \cdot (R_2^\top u_2) + \Omega_1 \cdot \tau_1 + \Omega_2 \cdot \tau_2
\end{aligned}$$

Substituting control terms from (17)-(20) we have

$$\dot{\mathcal{V}} = -k_{\Omega_1}\|\Omega_1\|^2 - k_{\Omega_2}\|\Omega_2\|^2 - (k_{v_1} + k_{v_2})\|v_1 - v_2\|^2 \leq 0$$

We have $\mathcal{V}$ bounded from below and $\dot{\mathcal{V}} \leq 0$, which gives that $\lim_{t\to\infty} \mathcal{V}(t)$ exists by monotonicity. Now by LaSalle's invariance principle, system dynamics converge asymptotically to the largest positively invariant set in

$$E = \left\{(R_1, \Omega_1, R_2, \Omega_2, r_1, v_1, r_2, v_2) | \dot{\mathcal{V}} = 0\right\} \tag{26}$$

from which we have $\Omega_1, \Omega_2 \to 0$ and $(v_1 - v_2) \to 0$ as $t \to \infty$, and $r_1 - r_2$, R_1, R_2 asymptotically goes to largest invariant set in E. Largest invariant set in E is given by $M = \{(R_1, \Omega_1, R_2, \Omega_2, r_1, v_1, r_2, v_2)\} \subset E$ which satisfies conditions

$$\widehat{b}_{21} \times \widehat{b}_{12} = 0 \tag{27}$$

$$R_1\left(k_2(d - d_0)\hat{b}_{12} - \frac{k_1}{d}\big(\hat{b}_{21} - (\hat{b}_{12} \cdot \hat{b}_{21})\hat{b}_{12}\big)\right) = 0 \tag{28}$$

$$R_2\left(k_2(d - d_0)\hat{b}_{21} - \frac{k_1}{d}\big(\hat{b}_{12} - (\hat{b}_{12} \cdot \hat{b}_{21})\hat{b}_{21}\big)\right) = 0 \tag{29}$$

From (27) we have $\widehat{b}_{12} = \pm\widehat{b}_{21}$. Consider the case where $\widehat{b}_{12} = \widehat{b}_{21}$, we have $\widehat{b}_{12} \cdot \widehat{b}_{21} = 1$, substituting in (28) and (29) we get

$$R_1\left(k_2(d - d_0)\hat{b}_{21} - \frac{k_1}{d}\big(\hat{b}_{21} - \hat{b}_{21}\big)\right) = 0 \quad \implies k_2(d - d_0)R_1\hat{b}_{21} = 0 \implies d = d_0$$

$$R_2\left(k_2(d - d_0)\hat{b}_{21} - \frac{k_1}{d}\big(\hat{b}_{21} - \hat{b}_{21}\big)\right) = 0 \quad \implies k_2(d - d_0)R_2\hat{b}_{21} = 0 \implies d = d_0$$

Similarly for the second case where $\widehat{b}_{12} = -\widehat{b}_{21}$, we have $\widehat{b}_{12} \cdot \widehat{b}_{21} = -1$, and

$$R_1 \left(k_2(d-d_0)(-\hat{b}_{21}) - \frac{k_1}{d}\big(\hat{b}_{21} - \hat{b}_{21}\big) \right) = 0 \quad \implies k_2(d-d_0)R_1(-\hat{b}_{21}) = 0 \implies d = d_0$$
$$R_2 \left(k_2(d-d_0)\hat{b}_{21} - \frac{k_1}{d}\big(-\hat{b}_{21} + \hat{b}_{21}\big) \right) = 0 \quad \implies k_2(d-d_0)R_2\hat{b}_{21} = 0 \implies d = d_0$$

Thus the largest invariant set in E is given by $M = Q_1 \cup Q_2$ where

$$Q_1 = \{(R_1, \Omega_1, R_2, \Omega_2, r_1, v_1, r_2, v_2) | \Omega_1 = \Omega_2 = 0, v_1 = v_2, \ \|r_1 - r_2\| = d_0, \\ R_1^T(r_1 - r_2) = R_2^T(r_1 - r_2)\} \qquad (30)$$
$$Q_2 = \{(R_1, \Omega_1, R_2, \Omega_2, r_1, v_1, r_2, v_2) | \Omega_1 = \Omega_2 = 0, v_1 = v_2, \ \|r_1 - r_2\| = d_0, \\ R_1^T(r_1 - r_2) = -R_2^T(r_1 - r_2)\} \qquad (31)$$

(ii) For the undesired configuration Q_2 given by $v_1 = v_2$, $\|r_1 - r_2\| = d$, $\Omega_1 = \Omega_2 = 0$ and $R_1^\top(r_2 - r_1) = -R_2^\top(r_1 - r_2)$, we have $\mathcal{V} = 2k_1$. Define

$$\mathcal{W} = 2k_1 - \mathcal{V} \qquad (32)$$

At undesired configuration Q_2, we have $\mathcal{W} = 0$. Now we can choose an arbitrarily close region to Q_2 (by choosing R_1, R_2 and $r_1 - r_2$) where the function $\mathcal{W} > 0$. Since $\dot{\mathcal{W}} = -\dot{\mathcal{V}} > 0$, there exists at any arbitrarily small neighbourhood of the undesired equilibrium, a solution trajectory that will escape, which gives that undesired equilibrium is unstable ([8], Theorem 3.3).

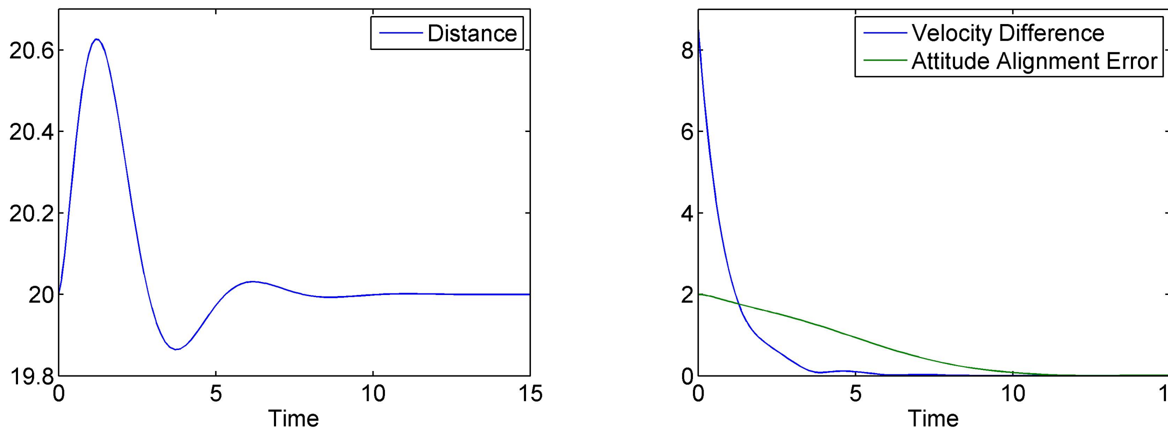
(iii) When conditions in (23)-(24) are satisfied, $\mathcal{V}(0) < 2k_1$. Since $\dot{\mathcal{V}} \leq 0$, we have

$$0 \leq \mathcal{V}(t) < \mathcal{V}(0) < 2k_1 \qquad (33)$$

This guarantees that the undesired equilibrium configuration is avoided and system dynamics converge to desired configuration Q_1.

□

Undesired equilibrium configuration may have stable manifolds to itself. But the set of all stable manifolds of undesired equilibrium configuration will have dimension less than the tangent space of the configuration space and thus will be of measure zero. Now compared to [6], the current work makes use of no third body and considers relative dynamics between the two spacecraft. Not having a LOS unit vectors to a third body compromises complete attitude alignment. But the proposed control law achieves attitude alignment along inertial LOS direction. This means that payloads of the two spacecrafts can point at same direction along inertial LOS direction, which is an important requirement in missions such as two spacecraft space telescope. Additionally the proposed control law controls the relative translational dynamics of the spacecraft along with rotational dynamics.



(a) Distance between two sopacecrafts

(b) Velocity alignment error fiunction Ψ_3 and attitude alignment error function Ψ_1

Figure 2: The error functions

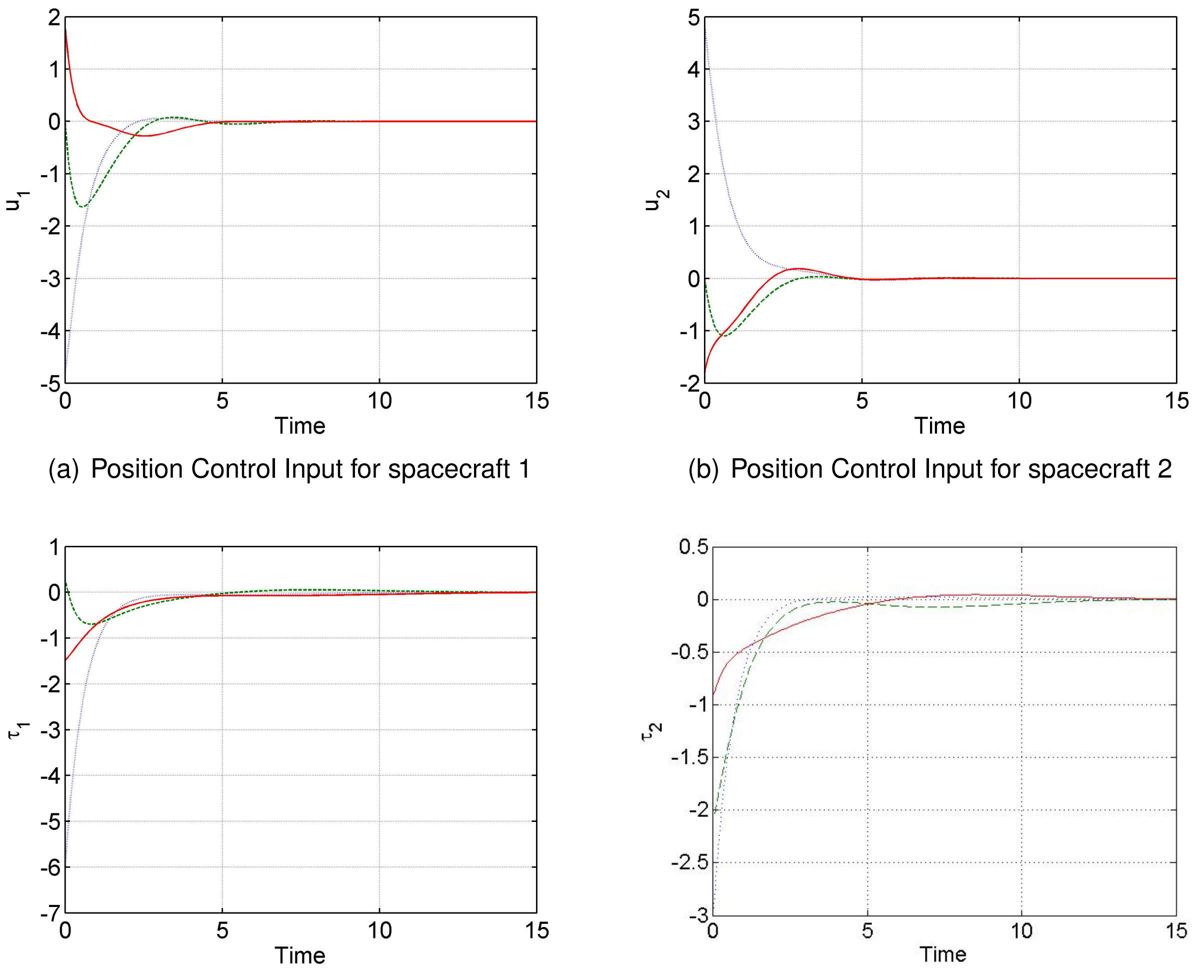
4 Simulation Results

Numerical simulation is carried out with following initial conditions. Moments of inertia of the two spacecraft are taken to be $J_1 = J_2 = \mathrm{diag}[2,\ 3,\ 5]$Nm^2. Let $a = \frac{1}{\sqrt{14}}[1,\ 2,\ 3]^\top$, and $\grave{a} = \frac{1}{\sqrt{3}}[1,\ 1,\ -1]^\top$. Following initial conditions are chosen for simulation: $r_1(0) = [0,\ 0,\ 0]^\top$, $v_1(0) = [3,\ 5,\ 8]^\top$ m/s, $\Omega_1(0) = [2,\ -0.1,\ 0.5]^\top$ rad/s, and $R_1(0) = \exp(\pi \mathsf{S}(a))$ where S(.) is the map from S(.) : $\mathbb{R}^3 \to so(3)$ defined in section 2 and $\exp$ is the matrix exponential. Initial conditions for spacecraft 2 $r_2(0) = r_1(0) + 10(a)$, $v_2(0) = [-5,\ 5,\ 11]^\top$ m/s, $\Omega_2(0) = [1,\ 0.7,\ 0.3]^\top$ rad/s and $R_2(0) = \exp(0.99\pi \mathsf{S}(\grave{a}))$. Also let the desired distance between the two spacecraft to be $d_0 = 20$ m. Initial conditions are chosen to be close to undesired equilibrium configuration. The control gains are chosen to be $k_{\Omega_1} = k_{\Omega_2} = 3$, $k_{v_1} = k_{v_2} = 0.6$, $k_1 = 0.5$, $k_2 = 1$. Figure (2) shows distance between satellites achieving desired value and velocity synchronization error and alignment error going to zero. The control inputs are shown in figures (3).

5 Conclusion

A combined position and attitude control law to achieve desired relative distance between the two spacecraft, velocity synchronization and attitude alignment about the line joining the two spacecraft is proposed. The control law makes use of only relative information and is shown that the control law achieves desired configuration asymptotically from all initial conditions except from a set of measure zero. The control law is demonstrated by a numerical example.

This work can be extended in several new directions. One of the future scope is to extend the control law to multi spacecraft formation where complete attitude alignment among spacecraft and formation keeping making use of LOS unit vector measurements



(a) Position Control Input for spacecraft 1

(b) Position Control Input for spacecraft 2

(c) Attitude Control Input for spacecraft 1

(d) Attitude Control Input for spacecraft 2

Figure 3: The control inputs applied to the two spacecraft. (First component: dotted, second component: dashed, third component: solid)

is desired. The idea of using LOS unit vectors for combined relative attitude and position control can be applied to tracking control and even swarm keeping control. Another future direction will be attempting similar control methodology, considering gravitational affects on spacecraft dynamics.

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References

[1] Scharf, D.P. and Hadaegh, F.Y. and Ploen, S.R., *A Survey of Spacecraft Formation Flying Guidance and Control(Part I): Guidance*, Proceedings of the American Control Conference, IEEE, Piscataway, NJ, June 2003, pp. 1733–1739.

[2] Scharf, D.P. and Hadaegh, F.Y. and Ploen, S.R., *A Survey of Spacecraft Formation Flying Guidance and Control(Part II): Control.* Proceedings of the American Control Conference, vol. 4, 2004, IEEE, pp. 2976-2985.

[3] Woffinden, David C., and Geller, David K. *Observability criteria for angles-only navigation.* Aerospace and Electronic Systems, IEEE Transactions on, Vol 45, No. 3, 2009, pp. 1194-1208.

[4] Gurfil, Pini, and Mishne, D. *Cyclic spacecraft formations: relative motion control using line-of-sight measurements only.* Journal of guidance, control, and dynamics, Vol.30, No. 1, 2007, pp. 214-226.

[5] Andrle, M.S. and Crassidis, J.L. and Linares, R. and Cheng, Y. and Hyun, B. *Deterministic relative attitude determination of three-vehicle formations*, Journal of Guidance, Control, and Dynamics, Vol. 32, No. 4, 2009, pp. 1077–1088.

[6] Wu, T.H. and Lee, T., *Vision-Based Relative Attitude Formation Tracking*, arXiv preprint arXiv:1209.2932 2012.

[7] Lee, T., *Relative attitude control of two spacecraft on SO (3) using line-of-sight observations*, American Control Conference (ACC), 2012, IEEE, pp. 167–172.

[8] Khalil, H.K., *Nonlinear systems* 2nd Edition, Ed. Prentice Hall, 1996.

[9] Bullo, F. and Murray, R.M. and Sarti, A., *Control on the sphere and reduced attitude stabilization*, California Institute of Technology, 1996.

[10] Ren, W., *Formation keeping and attitude alignment for multiple spacecraft through local interactions*, AIAA Journal of guidance, control, and dynamics, Vol. 30, No. 2, 2007, pp. 633–638.

[11] Alfriend, K.T. and Gurfil, P. and Vadali, S.R., *Spacecraft formation flying: dynamics, control, and navigation*, Butterworth-Heinemann, Elsevier Astrodynamics series, Vol 2, 2009.

A Calculations

Derivative of distance function

We have $d = \|r_1 - r_2\|$. For $d \neq 0$, we have

$$\begin{aligned}
\frac{d}{dt}\|d\| &= \frac{d}{dt}\sqrt{(r_2 - r_1)\cdot(r_2 - r_1)} = \frac{(r_2 - r_1)\cdot(v_2 - v_1)}{d} \\
&= \frac{1}{2d}\left[\left(R_1^\top(r_2 - r_1)\cdot R_1^\top(v_2 - v_1)\right) + \left(R_2^\top(r_1 - r_2)\cdot R_2^\top(v_1 - v_2)\right)\right] \\
&= \frac{1}{2}(\hat{b}_{12}\cdot \mathbf{v}_{21} + \hat{b}_{21}\cdot \mathbf{v}_{21})
\end{aligned} \tag{A.1}$$

We have made use of (10) for representing the derivative in a nice way.

Derivative of LOS unit vectors

We have $\hat{b}_{ij} = R_i^\top \frac{(r_j - r_i)}{d}$

$$\begin{aligned}
\frac{d}{dt}(\hat{b}_{ij}) &= \dot{R}_i{}^\top \frac{(r_j - r_i)}{d} + \frac{1}{d}R_i^\top(v_j - v_i) - R_i^\top(r_j - r_i)\frac{1}{2d^2}(\hat{b}_{12}\cdot \mathbf{v}_{21} + \hat{b}_{21}\cdot \mathbf{v}_{21}) \\
&= -R_i^\top \dot{R}_i R_i^\top(r_j - r_i) + \frac{1}{d}\mathbf{v_{ij}} - \frac{1}{2d}(\hat{b}_{12}\cdot \mathbf{v}_{21} + \hat{b}_{21}\cdot \mathbf{v}_{21})\hat{b}_{ij} \\
&= \hat{b}_{ij}\times \Omega_i + \frac{1}{d}\mathbf{v_{ij}} - \frac{1}{2d}(\hat{b}_{12}\cdot \mathbf{v}_{21} + \hat{b}_{21}\cdot \mathbf{v}_{21})\hat{b}_{ij}
\end{aligned} \tag{A.2}$$

Derivative of Ψ_1

We have $\Psi_1 = \frac{1}{2}\|\widehat{b}_{12} + \widehat{b}_{21}\|^2 = 1 + \widehat{b}_{12}\cdot \widehat{b}_{21}$. Time derivative is given by

$$\begin{aligned}
\dot{\Psi}_1 &= \widehat{b}_{21}\cdot\left(\hat{b}_{12}\times \Omega_1 + \frac{1}{d}\mathbf{v_{12}} - \frac{1}{2d}(\hat{b}_{12}\cdot \mathbf{v}_{21} + \hat{b}_{21}\cdot \mathbf{v}_{21})\hat{b}_{12}\right) \\
&\quad + \widehat{b}_{12}\cdot\left(\hat{b}_{21}\times \Omega_2 + \frac{1}{d}\mathbf{v_{21}} - \frac{1}{2d}(\hat{b}_{12}\cdot \mathbf{v}_{21} + \hat{b}_{21}\cdot \mathbf{v}_{21})\hat{b}_{21}\right) \\
&= (\hat{b}_{21}\times \hat{b}_{12})\cdot \Omega_1 + (\hat{b}_{12}\times \hat{b}_{21})\cdot \Omega_2 + \frac{1}{d}\hat{b}_{21}\cdot \mathbf{v_{12}} + \frac{1}{d}\hat{b}_{12}\cdot \mathbf{v_{21}} \\
&\quad - \frac{1}{d}(\hat{b}_{12}\cdot \hat{b}_{21})\left(\hat{b}_{12}\cdot \mathbf{v}_{21} + \hat{b}_{21}\cdot \mathbf{v}_{21}\right)
\end{aligned} \tag{A.3}$$