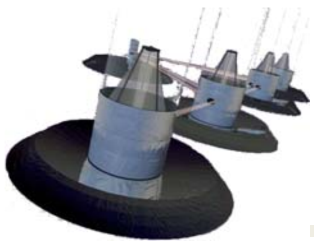

On Separation Principle for the Distributed Estimation and Control of Formation Flying Spacecraft

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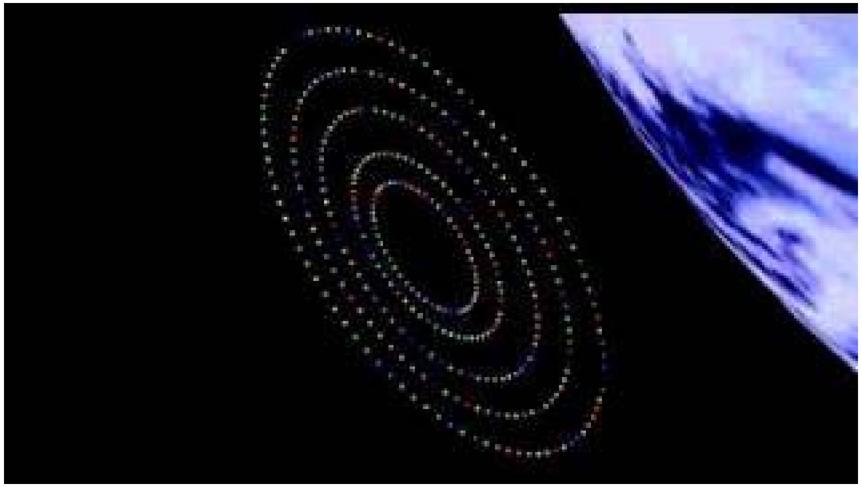
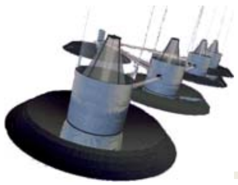
Outline

- Motivation
- Separation Principle (single system)
- Architecture of decentralized estimation and control
- A decentralized algorithm for control of formation flying spacecraft
- Concluding remarks

Motivation

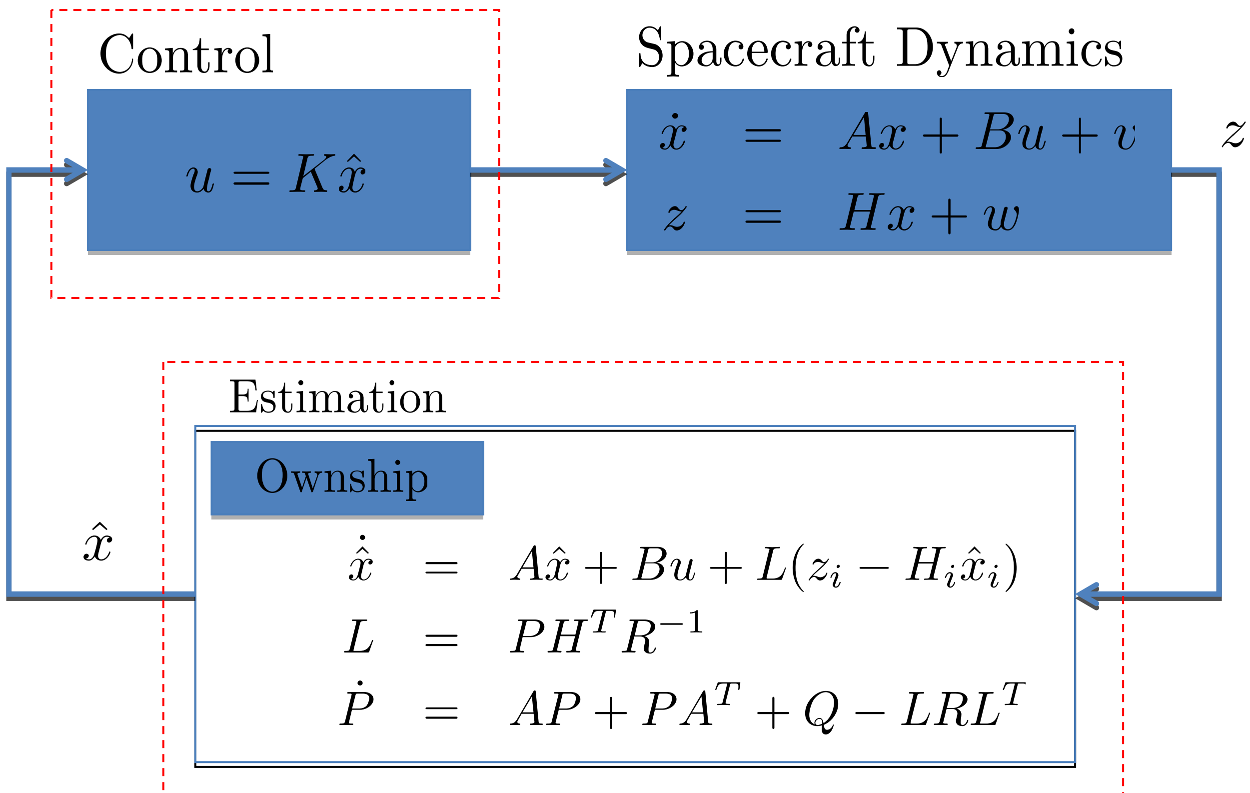
- Successful operation of swarms of femtosat rely on efficient yet fully decentralized estimation and control algorithms based on only local measurement and local communication.



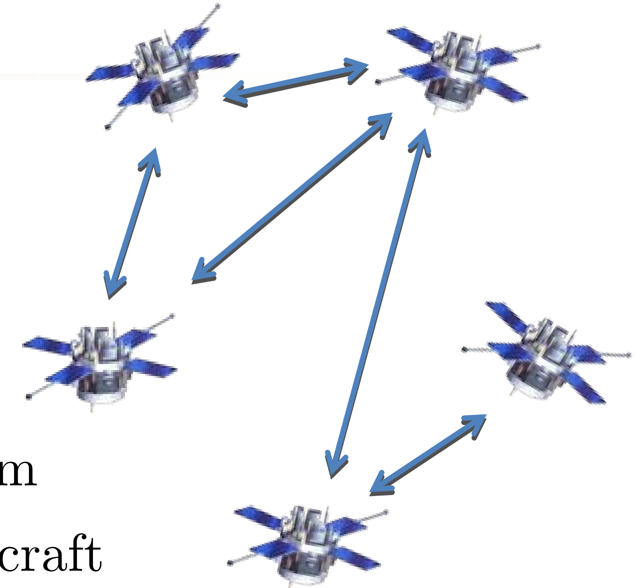


Separation Principle

Control and estimation blocks are separately designed yet the combination is stable.



Formation Control



Formation estimation and control strategies:

- Centralized

  Treat the formation as one giant system

  Not scalable for large number of spacecraft

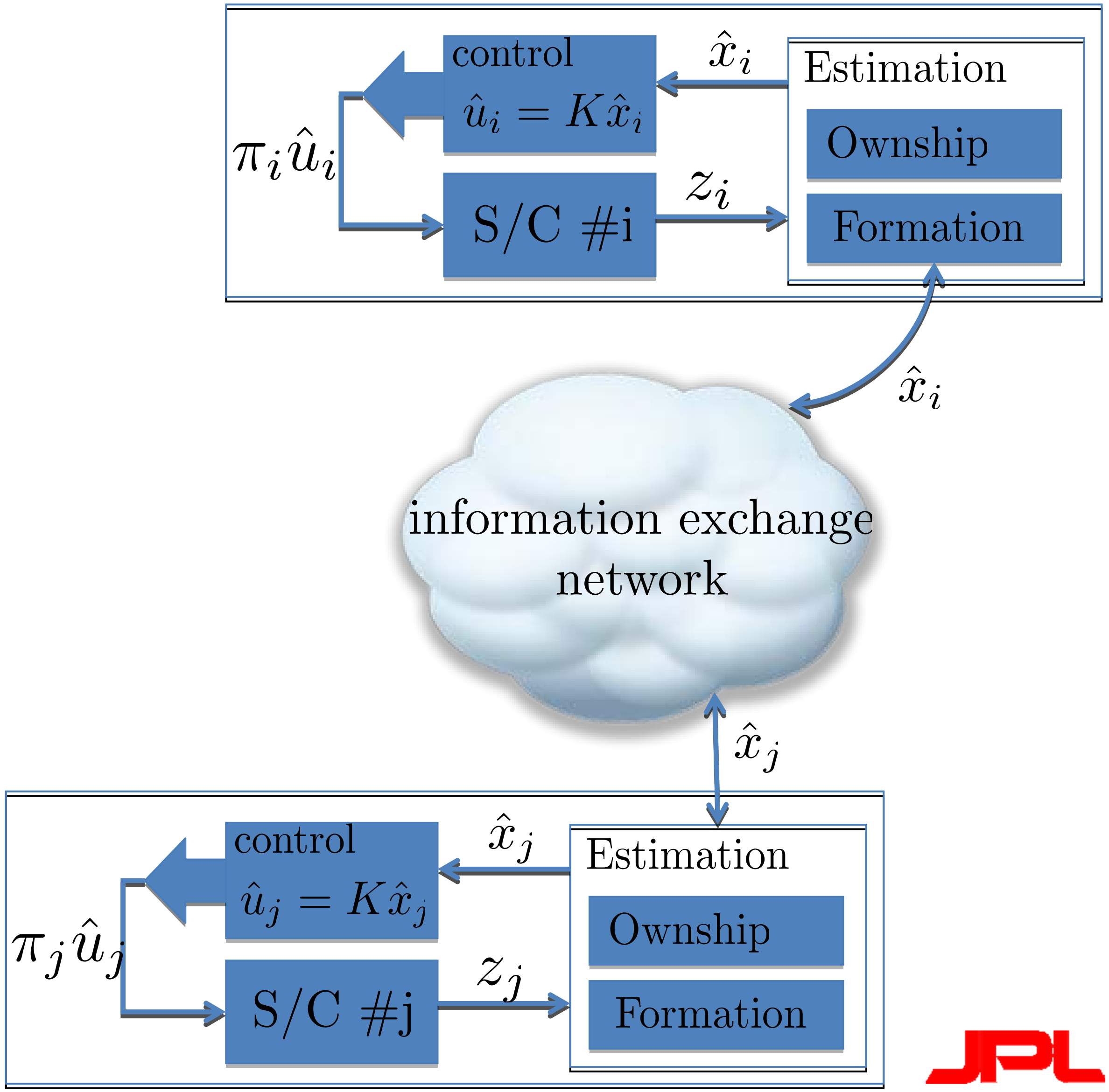
  States of the whole formation should be observable

- Decentralized

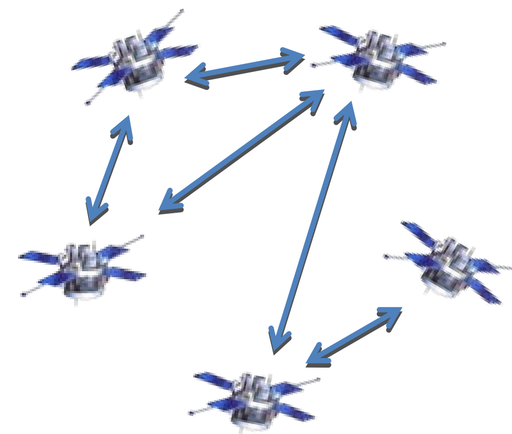
  Scalable and robust for large number of spacecraft

  Many unknowns! What is the best method?

Decentralized Control



Decentralized Estimation (No Control)



Consider a process (combined dynamics of all spacecraft)

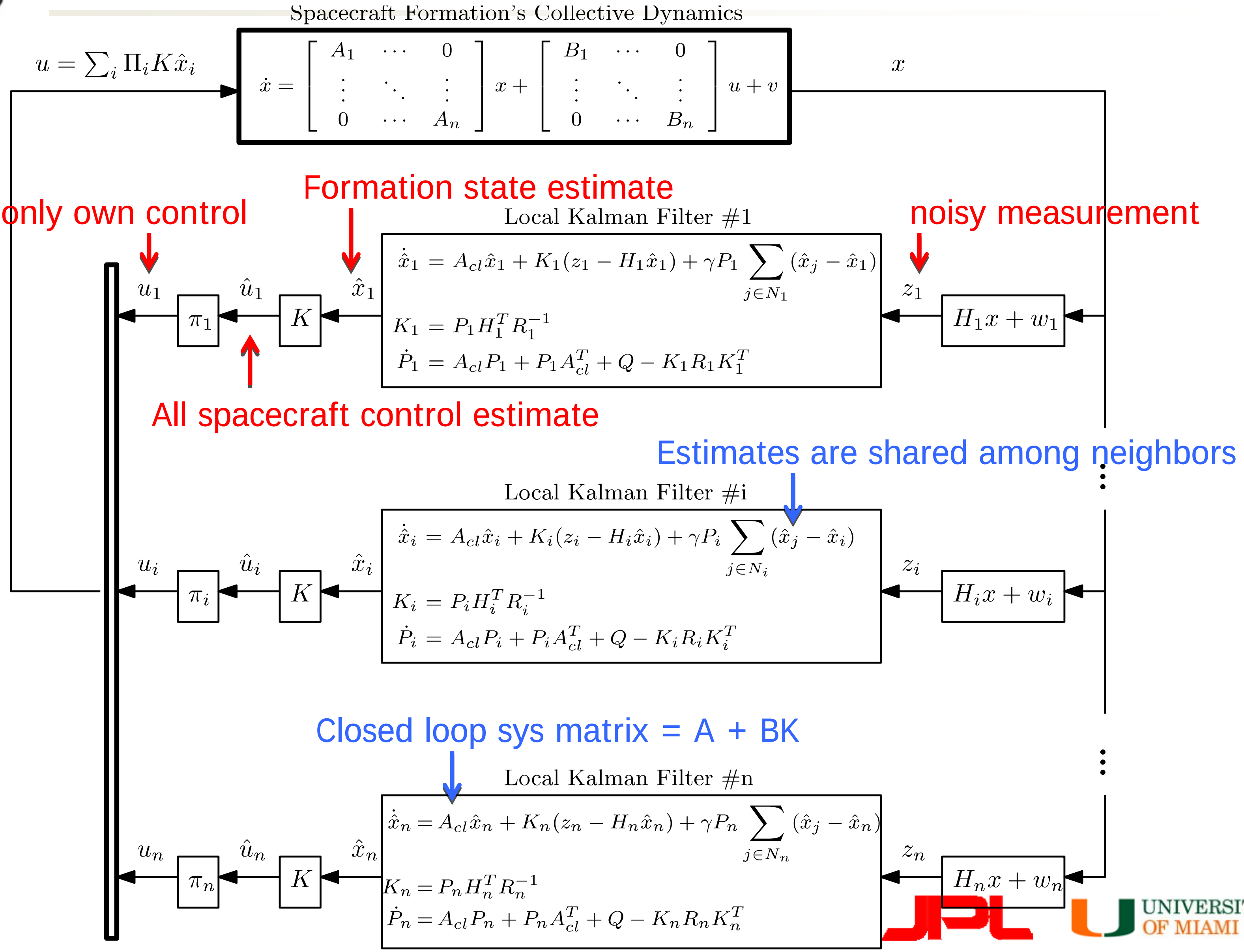
$$\dot{x} = Ax + v$$

with each spacecraft measurements as $z_i = H_i x + w_i$.

$$\begin{aligned}
\dot{\hat{x}}_i &= A\hat{x}_i + K_i(z_i - H_i\hat{x}_i) \boxed{+ \gamma P_i \sum_{j \in N_i} (\hat{x}_j - \hat{x}_i)} \\
K_i &= P_i H_i^T R_i^{-1} \\
\dot{P}_i &= AP_i + P_i A^T + Q - K_i R_i K_i^T
\end{aligned}$$

The estimation error, $\eta_i(t) = x(t) - \hat{x}_i(t)$, is asymptotically stable with Lyapunov function $V(\eta) = \sum_i \eta_i^T P_i^{-1} \eta_i$, i.e. as $t \to \infty$, $x = \hat{x}_i$.

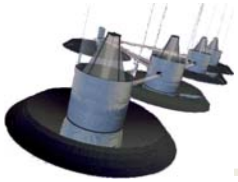
Proposed decentralized control



Does separation principle hold?

Proof: Define $\bar{\eta} = \begin{bmatrix} x \\ \eta_1 \\ \vdots \\ \eta_N \end{bmatrix}$ then $\dot{\bar{\eta}} = (M_1 - M_2 + M_3)$

$$
\begin{aligned}
M_1 &= \begin{bmatrix} A+BK & & & \\ & A+BK-K_1H_1 & & \\ & & \ddots & \\ & & & A+BK-K_nH_n \end{bmatrix}, \\
M_2 &= \begin{bmatrix} I \\ \vdots \\ I \end{bmatrix} \begin{bmatrix} \mathbb{0} & BK\Pi_1 K & \cdots & BK\Pi_N K \end{bmatrix}, \\
M_3 &= \begin{bmatrix} \mathbb{0} & \\ & L(\mathcal{G}) \otimes I_N \end{bmatrix}.
\end{aligned}
$$



Proof (cont.)

Lyapunov function

$$V(t) = \bar{\eta}^T \begin{bmatrix} \frac{1}{2}I & & & \\ & P_1^{-1} & & \\ & & \ddots & \\ & & & P_n^{-1} \end{bmatrix} \bar{\eta} = \sum_i \eta_i^T P_i^{-1} \eta_i + \frac{1}{2} x(t)^T x(t) \succ 0.$$

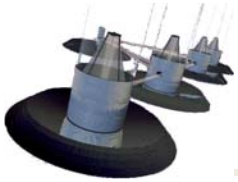
Closed loop and estimation error Dynamics

$$\begin{aligned} \dot{x} = Ax + Bu &= Ax + B \sum_i \Pi_i K \hat{x}_i \\ &= (A + BK)x - B \sum \Pi_i K \eta_i; \end{aligned}$$

$$\dot{\eta}_i = \dot{x} - \dot{\hat{x}}_i = (A + BK - K_i H_i)\eta_i + \gamma P_i \sum_{j \in \mathcal{N}_i} (\eta_j - \eta_i) - B \sum_{j=1}^{n} \Pi_j K \eta_j.$$

Derivative of the Lyapunov function

$$\dot{V}(t) = \sum_i \left(\eta_i^T P_i^{-1} \dot{\eta}_i + \dot{\eta}_i^T P_i^{-1} \eta_i - \eta_i^T P_i^{-1} \dot{P}_i P_i^{-1} \eta_i \right) + x^T \dot{x}.$$



Proof (cont.)

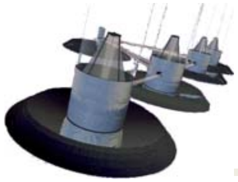
Define: $\Lambda_i = H_i^T R_i^{-1} H_i + P_i^{-1} Q P_i^{-1} \succ 0$,

negative definite

$$
\begin{aligned}
\dot{V} = & \boxed{-\quad \bar{\eta}^T \begin{bmatrix} -A-BK & & & \\ & \Lambda_1 & & \\ & & \ddots & \\ & & & \Lambda_n \end{bmatrix} \bar{\eta} - 2\gamma\eta^T \mathcal{L}\eta} \\
& -\quad \bar{\eta}^T \begin{bmatrix} 0 & B\Pi_1 K & B\Pi_2 K & \cdots & B\Pi_n K \\ 0 & P_1^{-1}B\Pi_1 K & P_1^{-1}B\Pi_2 K & \cdots & P_1^{-1}B\Pi_n K \\ 0 & P_2^{-1}B\Pi_1 K & P_2^{-1}B\Pi_2 K & \cdots & P_2^{-1}B\Pi_n K \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & P_n^{-1}B\Pi_1 K & P_n^{-1}B\Pi_2 K & \cdots & P_n^{-1}B\Pi_n K \end{bmatrix} \bar{\eta} \\
& -\quad \eta^T \begin{bmatrix} K^T\Pi_1 B^T P_1^{-1} & K^T\Pi_1 B^T P_2^{-1} & \cdots & K^T\Pi_1 B^T P_n^{-1} \\ K^T\Pi_2 B^T P_1^{-1} & K^T\Pi_2 B^T P_2^{-1} & \cdots & K^T\Pi_2 B^T P_n^{-1} \\ \vdots & \vdots & & \vdots \\ K^T\Pi_n B^T P_1^{-1} & K^T\Pi_n B^T P_2^{-1} & \cdots & K^T\Pi_n B^T P_n^{-1} \end{bmatrix} \eta.
\end{aligned}
$$

what about these terms?

Similarity transformation!



Proof (cont.)

Using proper similarity transformations:

$$\begin{bmatrix} I & & & -P_n \\ & P_1 & & -P_n \\ & & \ddots & \vdots \\ & & & P_n \end{bmatrix} \mathcal{M}_3 S^{-1} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & B\Pi_1 K P_1^{-1} & B\Pi_2 K P_2^{-1} & \cdots & \boxed{\sum_i B\Pi_i K P_i^{-1}} \end{bmatrix},$$

and

$$\begin{bmatrix} P_1^{-1} & & & \\ & P_2^{-1} & & \\ & & \ddots & \\ -P_1^{-1} & -P_2^{-1} & \cdots & P_n^{-1} \end{bmatrix} \mathcal{M}_4 \bar{S}^{-1} = \begin{bmatrix} 0 & 0 & \cdots & P_1^{-1} K^T \Pi_1 B^T \\ 0 & 0 & \cdots & P_2^{-1} K^T \Pi_2 B^T \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \boxed{\sum_i P_i^{-1} K^T \Pi_i B^T} \end{bmatrix}.$$

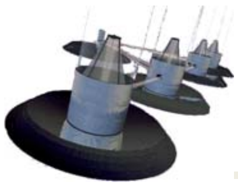
For closed loop system to be stable

$\sum_i B\Pi_i K P_i^{-1} \succ 0$ Note that : $\sum_i B\bar{\Pi}_i K = \bar{B} K$

$P_i^{-1} \succ 0$

Conclusions

- Decentralized formation estimation and control is the key to successful operation of swarms.
- Separation Principle is a fundamental concept in design of feedback controllers for systems, connecting the control and estimation.
- We proposed a consensus based estimation and control that rely on local interactions among agents and have more relaxed observability condition than previous works.
- Under certain conditions, proposed distributed estimation and control scheme is stable.



Thanks!