ON SEPARATION PRINCIPLE FOR THE DISTRIBUTED ESTIMATION AND CONTROL OF FORMATION FLYING SPACECRAFT

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Abstract: This work presents a distributed estimation and control architecture for formation flying spacecraft. We show how spacecraft can individually create a local estimate of the formation and create their own control action using that estimate. Proofs are provided to show that such a scheme is asymptotically stable and spacecraft will reach consensus on the formation states.

Keywords: Distributed Formation Control, Distributed Kalman Filter, Formation Estimation, Separation Principle.

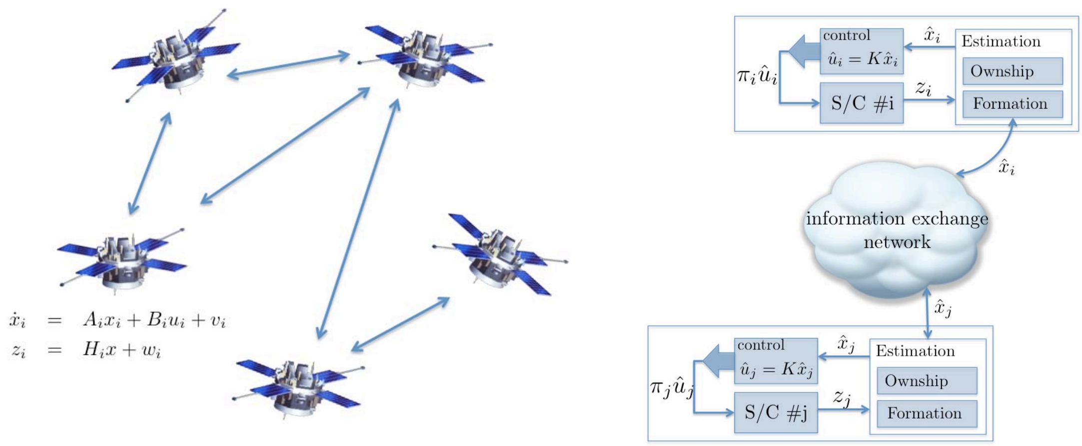
1. Introduction

Distributed space systems are expected to enable us to carryout space missions perceived impossible under the current monolithic design. The distributed nature of these spacecraft allows us to launch missions that rely on coordination among smaller spacecraft, improving the overall system reliability. However, a number of technical challenges should be resolved before such systems can successfully function to their maximum potential, and as a coherent unit. A fundamental question is: *How should the control system for such a space system be designed?*

Scharf *et al.* provided a comprehensive survey on guidance and control techniques for formation flying spacecraft [6, 7]. In general, one approach is to use a centralized estimation and control scheme. While possible, this approach does not allow for easy transition for changes in information exchange network or number of spacecraft in formation. Alternatively, in the distributed estimation and control architecture, each spacecraft individually estimates its own and the formation states based on the information received from its neighbors. These estimated states are used to locally create the control signal used for each spacecraft.

A large class of distributed estimator and controllers rely on information exchange networks modeled as communication graphs. A survey by Garin and Schenato discusses some of the distributed control and estimation techniques based on the consensus algorithms on the interaction graphs [1]. Olfati-Saber has proposed a distributed Kalman Filtering scheme using consensus algorithm for a group of sensors that, collectively, estimate states of a process [4]. Olfati and Jalalkamali have extended these results to the case of mobile sensors who estimate the state of a moving target and in the process try to improve an information measure [5].

Rantzer focused on the case where several controllers with access to different measurements collectively control a system [9]. Smith and Hadaegh proposed a system of parallel observers with noisy communication links and show under certain measurement and communication constraints, formation will be stable [8]. Hong *et al.* studied a leader follower network where followers have to estimate the velocity of the leaders [2]. Yang *et al.*, on the other hand, considered estimating some formation statistics



(a) (b)

Figure 1: (a) Information exchange network in distributed spacecraft constitutes a graph (b) Proposed distributed estimation and control architecture.

as a measure for current shape of the formation and use these individually estimated statistics to locally generate controls which are applied to each agent [10].

The fundamental question is *What is the best and most versatile distributed estimation and control scheme for such system and can one be designed independent of the other?*

This work presents a distributed estimation and control architecture for formation flying spacecraft where each spacecraft updates an estimate of its own as well as the formation states based on its local observations and information received from its neighbors. In return these estimates are used to generate a local control that collectively will drive the spacecraft to the desired formation.

2. Problem formulation

Consider n spacecraft with linear dynamics

$$\dot{x}_i(t) \;=\; A_i x_i(t) + B_i u_i(t) + v_i(t), \tag{1}$$

where index $i \in \{1, \ldots, n\}$ and v is a zero mean Gaussian noise. Defining collective vectors of state, control, and process noise respectively as $x = \mathrm{col}\{x_1, x_2, \ldots, x_n\}$, $u = \mathrm{col}\{u_1, u_2, \ldots, u_n\}$, and $v = \mathrm{col}\{v_1, v_2, \ldots, v_n\}$. Process noise vector v is zero mean Gaussian with covariance Q. The collective dynamics of all spacecraft can be represented as a linear system of the form

$$\dot{x}(t) = Ax(t) + Bu(t) + v(t). \tag{2}$$

The state and control matrices are diagonal matrices comprised of the individual spacecraft dynamic matrices, i.e. $A = \mathrm{diag}\{A_1, A_2, \ldots, A_n\}$ and $B = \mathrm{diag}\{B_1, \ldots, B_n\}$.

In a centralized control framework, one can design a stabilizing feedback controller $u(t) = Kx(t)$ that result in desired formation. This can be achieved through a variety of techniques such as pole-placement and optimal control to name a few. Now, the challenge is to implement such control strategy in a decentralized manner. This problem can be divided into two parts: (i) Each spacecraft should be able to estimate the states of the whole formation; (ii) Each spacecraft should compute and implement its own control input only. Just like any monolithic system, the fundamental question is whether the *Separation Principle* still holds for such design; i.e. does these individually designed control and estimation subsystem work together to render the system stable and reach the goal of the control?

3. Distributed Estimation

Each Spacecraft is assumed to have sensors that can measure some of its own states as well as that of potentially some neighboring spacecraft. Zero mean Gaussian measurement noise w_i with covariance R_i is considered to corrupt the measurement, i.e.

$$z_i \;=\; H_i x + w_i. \tag{3}$$

For the collective system of spacecraft described by (2) and individual spacecraft measurements (3); Provided that the pair (A, H) with $H = [H_1^T \, H_2^T \, \ldots \, H_n^T]^T$ is observable we use the result of Olfati-Saber [4] to propose a distributed Kalman Filter with dynamics

$$\begin{aligned}
\dot{\hat{x}}_i &= (A + BK)\hat{x}_i + K_i(z_i - H_i\hat{x}_i) + \gamma P_i \sum_{j \in \mathcal{N}_i} (\hat{x}_j - \hat{x}_i), \\
K_i &= P_i H_i^T R_i^{-1}, \\
\dot{P}_i &= (A + BK)P_i + P_i(A + BK)^T + Q - K_i R_i K_i^T.
\end{aligned} \tag{4}$$

Here $\hat{x}_i$ represents the estimated estates of the whole formation by spacecraft i (not to be mistaken by x_i, which represents the states of spacecraft i and has a much smaller size), K_i is the optimal Kalman gain, and P_i represent the covariance of the estimation error with dynamics presented in third equation of (4).

It is assumed that some spacecraft share their estimate of the formation states $\hat{x}_i$ with other spacecraft through an information exchange network. This network can be represented by a graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$. The vertex set $\mathcal{V}$ is the index of satellite in formation and edge set $\mathcal{E}$ is the set of unordered pairs (i, j) of indices of communicating spacecraft. Two spacecrafts are called neighbors[1] if they share an edge, i.e. $(i, j) \in \mathcal{E} \Leftrightarrow j \in \mathcal{N}_i$. Figure 1(a) depicts how the information exchange network is presented as a graph with spacecraft as nodes and communication links as edges.

The last term of first equation drives the estimation dynamics toward consensus on the estimated states $\hat{x}_i$. Its contribution depends on the sum of the difference between own estimate and that received from its neighbors, as well as confidence in own estimate, represented by error covariance P_i, and a positive constant γ that selects how much weight should be given to the consensus versus the local Kalman filter dynamics.

In this design it is sufficient that the collection of all spacecraft measurements H and state matrix A be an observable pair. This condition is a much relaxed requirement,

[1] Not necessarily physical neighborhood, although in cases of limited power communication physical neighbors are the same as Δ-disk graph neighbors.

compared to the proposed parallel observers of [8] which requires all pairs (A, H_i) to be observable. Another advantage is that the current formulation use the knowledge of noise statistics and system dynamics to generate the provably most optimal estimate of the states.

4. Distributed Control

Recall that our goal is to generate a decentralized state feedback control based on the centralized synthesis of feedback gain K. A simple solution will be that each spacecraft generate its estimate of what the control of the entire formation should be based on their local estimate of entire formation states and the designed gain K, i.e. $\hat{u}_i(t) = K\hat{x}_i$. Since, each spacecraft only has authority over its own control inputs at any given time, each one can only implement the portion of control that is pertaining to them. We represent this with a projection π_i such that

$$u_i = \pi_i \hat{u}_i = \begin{bmatrix} \mathbb{0} & \cdots & I & \cdots & \mathbb{0} \end{bmatrix} \hat{u}_i, \tag{5}$$

where $\mathbb{0}$ and I represent zero and identity matrices of proper size respectively. Note that again $\hat{u}_i$ is the estimate of controls of the whole formation as perceived by spacecraft i, while a much lower dimension u_i is the implemented control by that spacecraft. Another handy projection definition $\Pi_i = \pi_i^T \pi_i$ is a partitioning of the identity matrix, i.e. $I = \sum_i \Pi_i$, where $[\mathbb{0}\ u_i^T\ \mathbb{0}]^T = \Pi_i \hat{u}_i$.

Hence the control of the whole formation (implemented locally at each spacecraft)

$$u(t) = \sum_i \Pi_i \hat{u}_i = \sum_i \Pi_i K \hat{x}_i. \tag{6}$$

Figure 2 illustrates the distributed estimation and control signal flow as described above.

5. Separation Principle

Question now is if the separation principle holds and if the feedback control designed for a centralized scheme can stabilize system (2) of formation flying spacecraft with measurements (3) and distribution estimation (4), implementing local controls synthesized as (5)?

Defining estimation error of each spacecraft as $\eta_i = x - \hat{x}_i$ and consider a quadratic Lyapunov function

$$V(t) = \sum_i \eta_i^T P_i^{-1} \eta_i + \frac{1}{2} x(t)^T x(t) \succ 0. \tag{7}$$

The closed loop dynamics under distributed control (6) is

$$\begin{aligned} \dot{x} = Ax + Bu &= Ax + B\sum_i \Pi_i K \hat{x}_i \\ &= (A + BK)x - B\sum_i \Pi_i K \eta_i; \end{aligned} \tag{8}$$

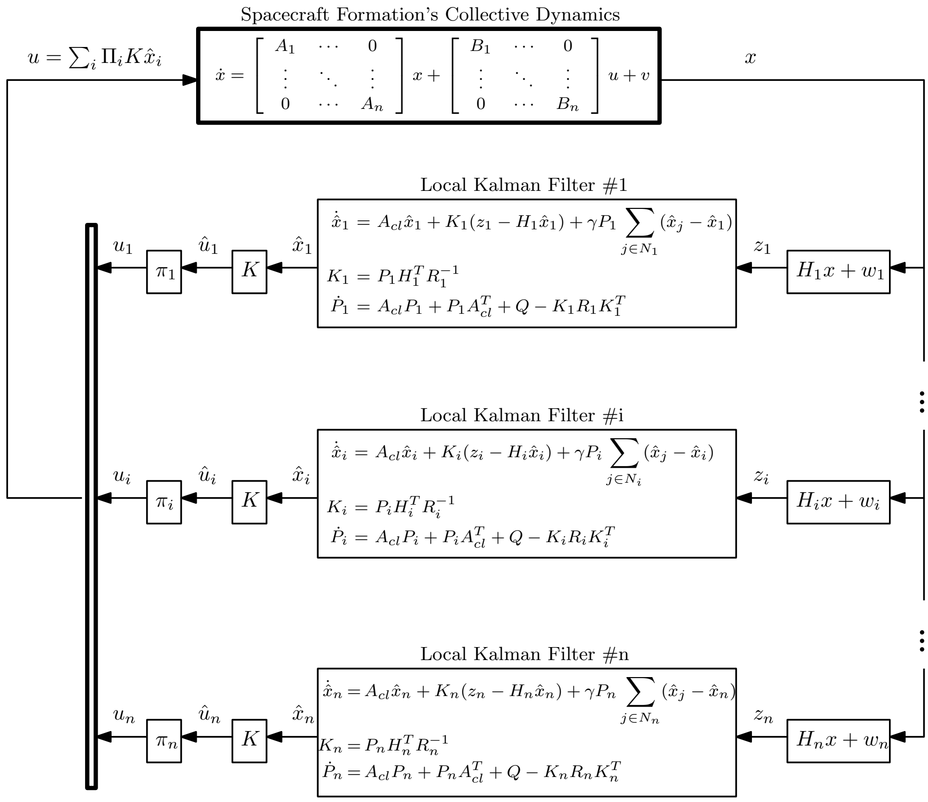


Figure 2: Schematics of the proposed distributed estimation and control.

While the error dynamics of each of the estimators, noting that $\hat{x}_j - \hat{x}_i = \eta_i - \eta_j$, is described by

$$\dot{\eta}_i = \dot{x} - \dot{\hat{x}}_i = (A + BK - K_i H_i)\eta_i + \gamma P_i \sum_{j \in \mathcal{N}_i} (\eta_j - \eta_i) - B \sum_{j=1}^{n} \Pi_j K \eta_j. \tag{9}$$

Derivative of the Lyapunov function is calculated as

$$\dot{V}(t) = \sum_i \left(\eta_i^T P_i^{-1} \dot{\eta}_i + \dot{\eta}_i^T P_i^{-1} \eta_i - \eta_i^T P_i^{-1} \dot{P}_i P_i^{-1} \eta_i \right) + x^T \dot{x}. \tag{10}$$

Noting that Covariance matrices P_i, Q, R_i are symmetric positive semi-definite, the above derivative is rewritten as

$$\begin{aligned} \dot{V}(t) = & - \sum_i \eta_i^T (H_i^T R_i^{-1} H_i + P_i^{-1} Q P_i^{-1}) \eta_i + x^T (A + BK) x - x^T B \sum_i \Pi_i K \eta_i \\ & - \sum_i \left(\eta_i^T P_i^{-1} B \sum_{j=1}^{n} \Pi_j K \eta_j + (\sum_{j=1}^{n} \eta_j^T K^T \Pi_j) B^T P_i^{-1} \eta_i \right) \\ & + 2\gamma \sum_i \sum_{j \in \mathcal{N}_i} \eta_i^T (\eta_j - \eta_i) \end{aligned} \tag{11}$$

Define $\Lambda_i = H_i^T R_i^{-1} H_i + P_i^{-1} Q P_i^{-1} \succ 0$, column vector of all estimation errors as $\eta = \mathrm{col}\{\eta_1, \eta_2, \ldots, \eta_n\}$ and augmented state and estimation errors of the formation as

$\bar{\eta} = [x^T \eta^T]^T$, the above equation is transformed to

$$
\begin{aligned}
\dot{V} = & - \bar{\eta}^T \begin{bmatrix} -A - BK & & & \\ & \Lambda_1 & & \\ & & \ddots & \\ & & & \Lambda_n \end{bmatrix} \bar{\eta} - 2\gamma \eta^T \mathcal{L} \eta \\
& - \bar{\eta}^T \begin{bmatrix} \mathbb{0} & B\Pi_1 K & B\Pi_2 K & \cdots & B\Pi_n K \\ \mathbb{0} & P_1^{-1} B\Pi_1 K & P_1^{-1} B\Pi_2 K & \cdots & P_1^{-1} B\Pi_n K \\ \mathbb{0} & P_2^{-1} B\Pi_1 K & P_2^{-1} B\Pi_2 K & \cdots & P_2^{-1} B\Pi_n K \\ \vdots & \vdots & \vdots & & \vdots \\ \mathbb{0} & P_n^{-1} B\Pi_1 K & P_n^{-1} B\Pi_2 K & \cdots & P_n^{-1} B\Pi_n K \end{bmatrix} \bar{\eta} \\
& - \eta^T \begin{bmatrix} K^T \Pi_1 B^T P_1^{-1} & K^T \Pi_1 B^T P_2^{-1} & \cdots & K^T \Pi_1 B^T P_n^{-1} \\ K^T \Pi_2 B^T P_1^{-1} & K^T \Pi_2 B^T P_2^{-1} & \cdots & K^T \Pi_2 B^T P_n^{-1} \\ \vdots & \vdots & & \vdots \\ K^T \Pi_n B^T P_1^{-1} & K^T \Pi_n B^T P_2^{-1} & \cdots & K^T \Pi_n B^T P_n^{-1} \end{bmatrix} \eta. \qquad (12)
\end{aligned}
$$

The first term in (12) is a diagonal matrix, $\mathcal{M}_1$, of positive definite matrices $\Lambda_i \succ 0$ and the negative of closed loop state matrix. By design, control gain K is chosen such that the closed loop system is stable, i.e. $A + BK \prec 0$. Hence the first term is positive definite. The second term is quadratic in $\mathcal{L} = L(\mathcal{G}) \otimes I_N$, where $L(\mathcal{G})$ represent the Laplacian of the information exchange graph $\mathcal{G}$ and I_N is identity matrix of the size $N = |x|$, size of vector x. As seen in [3], this matrix is positive semi-definite hence this term also is positive semi-definite, i.e. $-2\gamma\eta^T \mathcal{L}\eta \prec 0$. The next step is to find the eigenvalues of the two remaining matrices to determine the stability of the synthesized estimation and control scheme.

The first matrix can be reduces to a lower diagonal matrix using a similarity transformation

$$
S = \begin{bmatrix} I & & & -P_n \\ & P_1 & & -P_n \\ & & \ddots & \vdots \\ & & & P_n \end{bmatrix} \quad \text{and} \quad S^{-1} = \begin{bmatrix} I & & & I \\ & P_1^{-1} & & P_1^{-1} \\ & & \ddots & \vdots \\ & & & P_n^{-1} \end{bmatrix}.
$$

Similarly, a similarity transformation of the form

$$
\bar{S} = \begin{bmatrix} P_1^{-1} & & & \\ & P_2^{-1} & & \\ & & \ddots & \\ -P_1^{-1} & -P_2^{-1} & \cdots & P_n^{-1} \end{bmatrix} \quad \text{and} \quad \bar{S}^{-1} = \begin{bmatrix} P_1 & & & \\ & P_2 & & \\ & & \ddots & \\ -P_n & -P_n & \cdots & P_n \end{bmatrix},
$$

is used to upper diagonalize the last matrix of equation (12). We choose to use these matrices because eigenvalues of a matrix do not change under similarity transformations.

Hence,

$$S\mathcal{M}_3S^{-1} = \begin{bmatrix} \mathbb{0} & \mathbb{0} & \mathbb{0} & \cdots & \mathbb{0} \\ \mathbb{0} & \mathbb{0} & \mathbb{0} & \cdots & \mathbb{0} \\ \mathbb{0} & \mathbb{0} & \mathbb{0} & \cdots & \mathbb{0} \\ \vdots & \vdots & \vdots & & \vdots \\ \mathbb{0} & B\Pi_1KP_1^{-1} & B\Pi_2KP_2^{-1} & \cdots & \sum_i B\Pi_iKP_i^{-1} \end{bmatrix}, \tag{13}$$

and

$$\bar{S}\mathcal{M}_4\bar{S}^{-1} = \begin{bmatrix} \mathbb{0} & \mathbb{0} & \cdots & P_1^{-1}K^T\Pi_1B^T \\ \mathbb{0} & \mathbb{0} & \cdots & P_2^{-1}K^T\Pi_2B^T \\ \vdots & \vdots & & \vdots \\ \mathbb{0} & \mathbb{0} & \cdots & \sum_i P_i^{-1}K^T\Pi_iB^T \end{bmatrix}. \tag{14}$$

Eigenvalues of these lower (upper) diagonal matrices are represented by the eigenvalues of its diagonal blocks. Other than a number of zero eigenvalues these matrices have similar eigenvalues (one is the transpose of the other) represented by eigenvalues of the matrix $\sum_i B\Pi_iKP_i^{-1}$. Consequently, a sufficient condition for the proposed distributed control to be stable and for the separation principle to hold is that $\sum_i B\Pi_iKP_i^{-1}$ be positive semi-definite. Matrices P_i^{-1} are symmetric positive definite and $\sum_i B\Pi_iK = BK$. Under the above condition, derivative of the Lyapunov function is negative definite and system is asymptotically stable. The estimation errors η_i will reach zero, i.e. $\hat{x}_i = x$ (for more details see [3]).

It is noteworthy that the collective dynamics of all spacecraft states and their estimation error from (8) and (9) is

$$\dot{\bar{\eta}} = (M_1 + M_2 + M_3)\,\bar{\eta}, \tag{15}$$

where

$$\begin{aligned} M_1 &= \begin{bmatrix} A+BK & & & \\ & A+BK-K_1H_1 & & \\ & & \ddots & \\ & & & A+BK-K_nH_n \end{bmatrix}, \\ M_2 &= \begin{bmatrix} I \\ \vdots \\ I \end{bmatrix} \begin{bmatrix} \mathbb{0} & BK\Pi_1K & \cdots & BK\Pi_NK \end{bmatrix}, \\ M_3 &= \begin{bmatrix} \mathbb{0} & \\ & L(\mathcal{G})\otimes I_N \end{bmatrix}. \end{aligned}$$

The proposed Lyapunov function (7) is quadratic on $\bar{\eta}$, which evolves based on the above mentioned equation, i.e.

$$V(t) = \bar{\eta}^T \begin{bmatrix} \frac{1}{2}I & & & \\ & P_1^{-1} & & \\ & & \ddots & \\ & & & P_n^{-1} \end{bmatrix} \bar{\eta}.$$

6. Conclusions

We show that under certain conditions a feedback controller designed to stabilize a spacecraft formation can be used to generate control signals locally. Control inputs are generated locally using distributed estimation of full formation states based on local measurements and communication with some neighboring spacecraft. The condition for stability of the spacecraft formation calls for $\sum_i B\Pi_i K P_i^{-1}$ to be positive definite. This is the direct result of using a Lyapunov stability analysis on a system comprised of all spacecraft states and their estimation estates. Proper adjustments to the estimation filters can potentially relax the stability condition provided and is the topic of our future studies.

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