Coordinated 6-DOF Control of Dual Spacecraft Formation

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Needs of Formation Flying

- Degree of performance proportional to instrument size
- Payload distribution over a no. of satellites

- High Resolution
- Wider Swath
- Simultaneous coverage

- Modularity
- Launch Flexibility

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Investigations by Formation Flying

- SAR interferometry
- Planet Finding and Imaging
- Gravimetry for Gravitational Mapping
- **Stereo Imaging (addressed in this paper)**
- Space Weather Monitoring
- 3-D Mapping

Requirements of Stereo Imaging by Multi-Spacecrafts

- Relative position modeling of formations including perturbations
- GPS based attitude and Position Estimation for Constellation and Earth Orbit Formations
- So far researchers concentrated mainly on relative position estimation and control
- Very Little effort has gone in the field of Relative Attitude Control
- Position and Attitude Loops were analyzed in decoupled concepts
- Co-ordinated 6-dof control effort using modern control theory

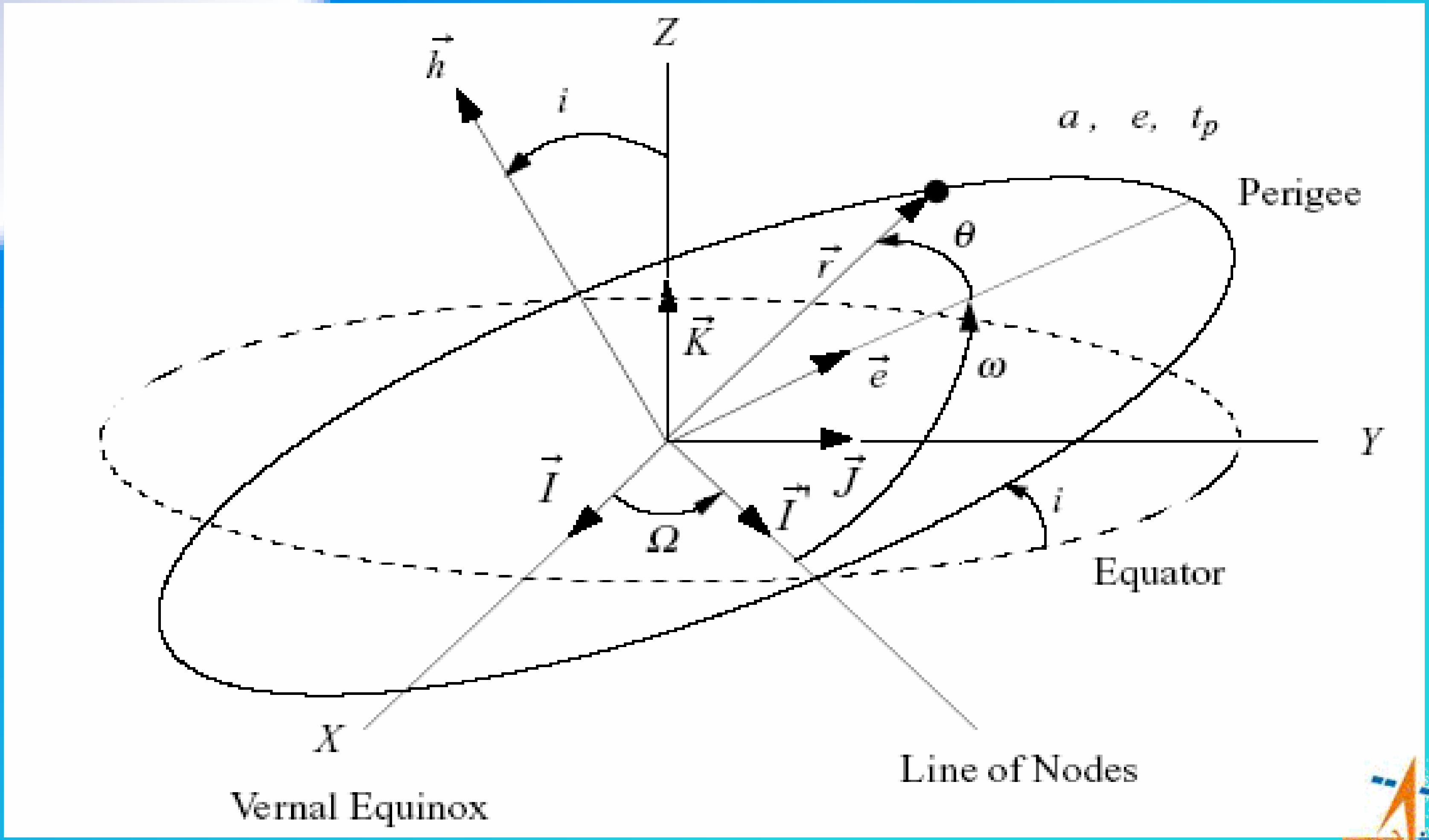
Formation Control Requirements

- Autonomous real-time control of relative position and relative attitude among formation members
  - Relative Attitude Maintenance is needed to orient the thrust direction for efficient reconfiguration of position loop
  - Data collection by formation cluster is a function of both spacecraft position and attitude
- Attenuate the effect of perturbing forces which otherwise disturbs the formation either by inter-satellite interference and collisions or by path divergence
- Performance trade-off between Mission Goals and Fuel cost.
- General guideline is to meet the baseline requirement of 'x', the control accuracy should be better than x/10 and the navigation or sensing accuracy should be better than x/100.

Contributions of the Paper

- An innovative coupled dynamics and control algorithm is developed for High Precision Stereo Imaging by a dual-microsatellite formation flying mission
- 6 Degree of Freedom model where each follower generates the attitude references in real-time, based on relative position and translational motion between the leader and its followers
- For the orbit control loop, the desired parameters are specified in terms of orbital elements and GPS + accelerometer forms the feedback source
- Desired attitude reference is derived on-board from the specified Earth view co-ordinates. Star tracker + gyro provides feedback information

Orbit Elements – Defined in ECI



Relative Orbit Elements (ROE)

$$\Delta\alpha = \begin{pmatrix} \Delta a \\ a_F \Delta e_X \\ a_F \Delta e_Y \\ a_F \Delta i_X \\ a_F \Delta i_Y \\ a_F \Delta u \end{pmatrix} = \begin{pmatrix} a_L - a_F \\ a_F\left(e_L \cos\omega_L - e_F \cos\omega_F\right) \\ a_F\left(e_L \sin\omega_L - e_F \sin\omega_F\right) \\ a_F\left(i_L - i_F\right) \\ a_F\left(\Omega_L - \Omega_F\right)\sin i_F \\ a_F\left(u_L - u_F\right) \end{pmatrix}$$

L: Leader s/c ; F: Follower s/c

ROE to Cartesian or Hill's Vector (Position, Velocity)

$$\begin{bmatrix} \Delta r_x \\ \Delta r_y \\ \Delta r_z \\ \Delta v_x \\ \Delta v_y \\ \Delta v_z \end{bmatrix} = \begin{bmatrix} 1 & -\cos u & -\sin u & 0 & 0 & 0 \\ -1.5(u-u_0) & 2\sin u & -2\cos u & 0 & \cot i & 1 \\ 0 & 0 & 0 & \sin u & \cos u & 0 \\ 0 & n\sin u & -n\cos u & 0 & 0 & 0 \\ -1.5n & 2n\cos u & 2n\sin u & 0 & 0 & 0 \\ 0 & 0 & 0 & n\cos u & n\sin u & 0 \end{bmatrix} \Delta\alpha$$

- u and u_0 are the mean argument of latitude at any time t and epoch time t_0
- 'n' is the mean orbital rate in rad/s

Relative Attitude Kinematics

The Relative Attitude of the Follower S/C with respect to Leader is given by

$$q = q_L^* \otimes q_F$$

Relative Attitude Modeling

$$\dot{q} = \frac{1}{2}\left(q \otimes \omega\right)$$

$$\omega = \omega_{L,F}^{F} = \omega_{I,F}^{F} + \omega_{L,I}^{F} = \omega_{I,F}^{F} - \omega_{I,L}^{F}$$

The notation $\omega_{A,B}^{C}$ indicates the angular velocity of frame 'B' with respect to frame 'A' as represented in frame 'C'.

Navigation System

- **IRAP (Inertial Reference Gyro with Accelerometer Package) with GPS and Star Tracker Updates forms the Navigation Loop**

Attitude Path

- Star Tracker provides the instantaneous attitude of the spacecraft with respect to Inertial Frame. Gyro data is used to propagate the attitude between star tracker updates.
- This provides the instantaneous attitude of the spacecraft with respect to the Inertial Frame which is used along with the gyro rate for the leader.
- For the follower spacecraft control, the relative attitude and rate of the follower with respect to the leader's orbit reference frame is represented in the follower frame

Navigation System (Contd…)

Position Path

- Accelerometer measures the acceleration in body frame which is represented in inertial frame using transformation matrix obtained from the instantaneous body to inertial attitude quaternion.
- This transformed acceleration is double integrated and used to propagate the position information between absolute GPS updates.
- The processed Cartesian data is transformed to instantaneous orbital elements and fed to the controller

Controller Design

- Linear Quadratic Controller (LQR) is used for both position and attitude control loops.
- This is based on optimization of a specified cost functional
- For the position Loop, fuel consumption is the cost functional as thruster is the prime actuator to maintain the relative position
- For attitude loop, time is the cost functional to be optimized to perform fast reorientations with wheels to capture the imaging spots in specified time and also to acquire the desired thruster orientation at the time specified for Orbit correction.

LQR Based Controller

The cost functional for the LQR controller is given by

$$J = \frac{1}{2}\int \left[x^T Q(t) x + u^T R(t) u \right] dt + \frac{1}{2} x^T (t_f) Q_t x(t_f)$$

Where, Q(t) is a positive semidefinite state weighting matrix, R(t) is a positive definite control weighting matrix and Q_t is positive definite terminal – weighting matrix. The objective is to find the feedback gain K which minimizes the cost functional J.

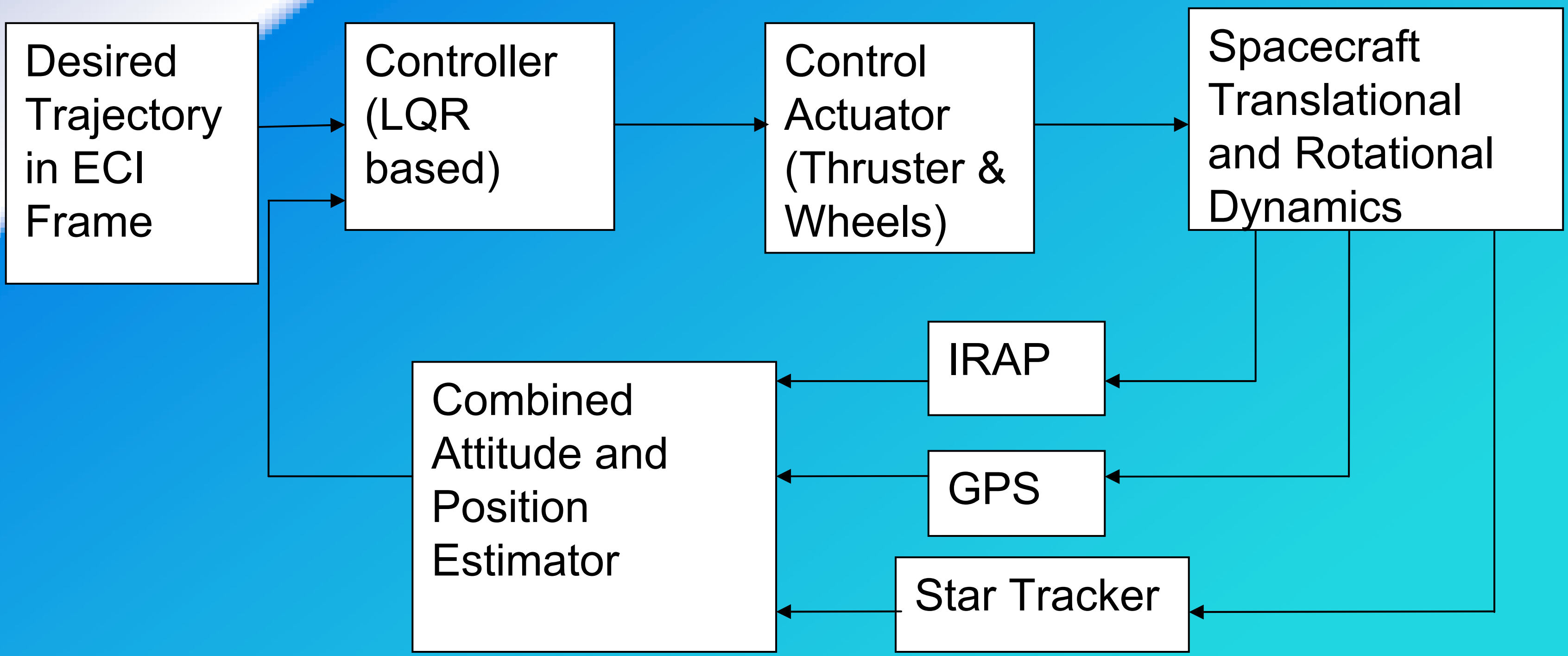
The optimum feedback gain is given by

$$K = R^{-1} B^T P(t)$$

The matrix P is computed by solving the continuous time Riccati equation

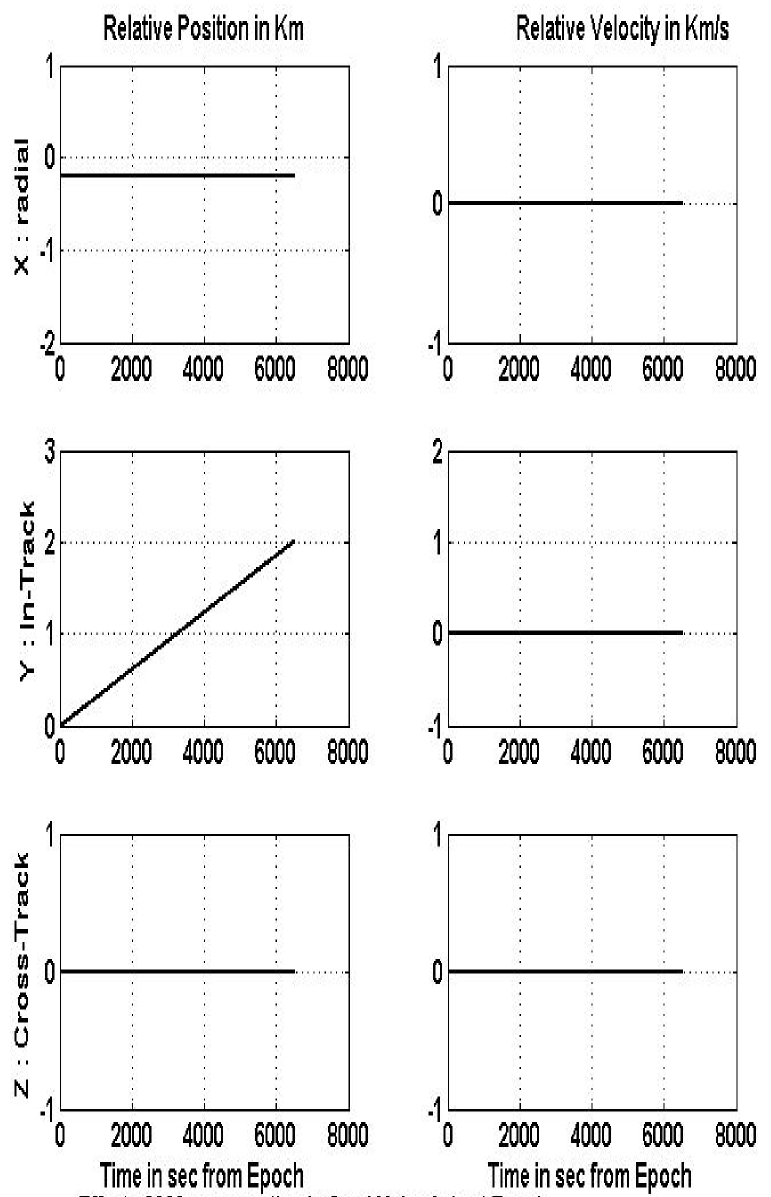
$$\dot{P}(t) + A^T P(t) + P(t) A - P(t) B R^{-1} B^T P(t) + Q = 0$$

Block Diagram of Satellite Formation Control Loop (Coupled Position and Attitude)

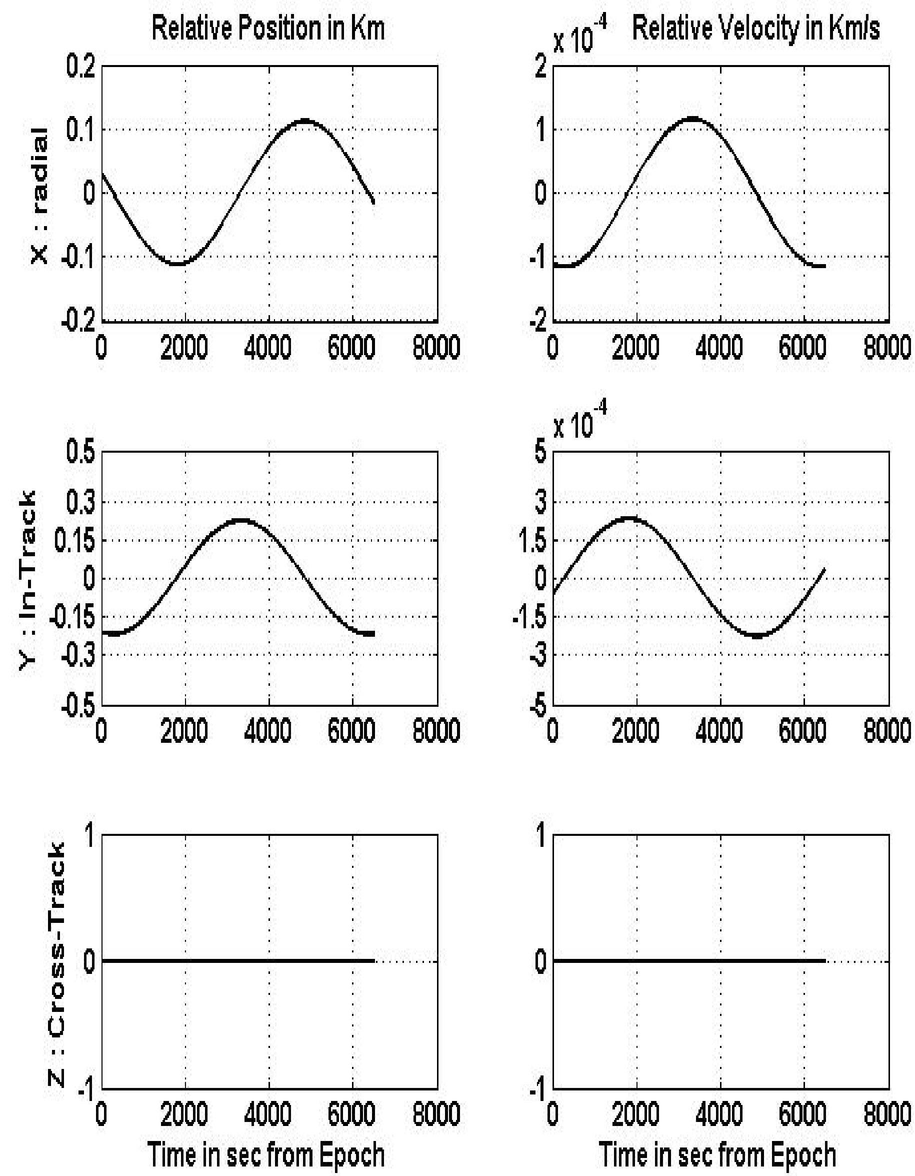


Effect of Initial Orbit Element Separation on S/C Relative Position and Velocity (Open Loop)

Effect of Initial Separation in ‘a’ and ‘e’ on Open Loop Position and Velocity

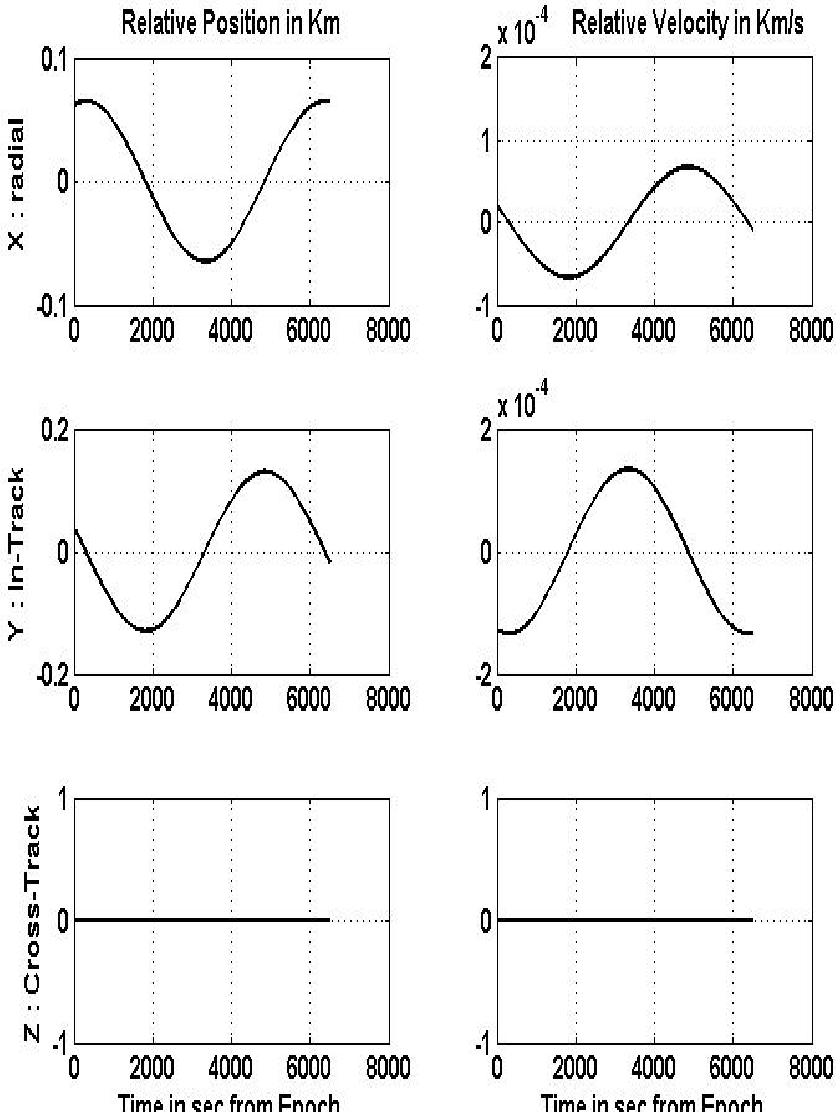


Effect of 200 m separation in Semi Major Axis at Epoch

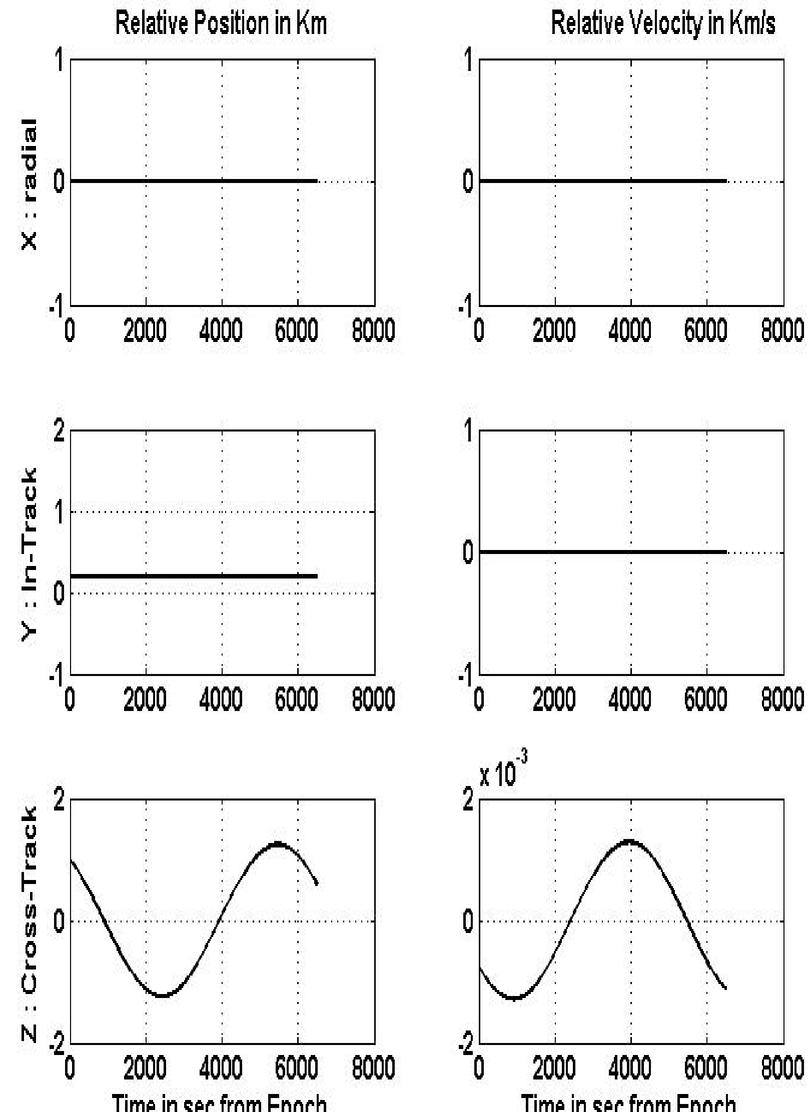


Effect of 3% m Variation in Eccentricity at Epoch

Effect of Initial Separation in ‘ω’ and ‘Ω’ on Open Loop Position and Velocity

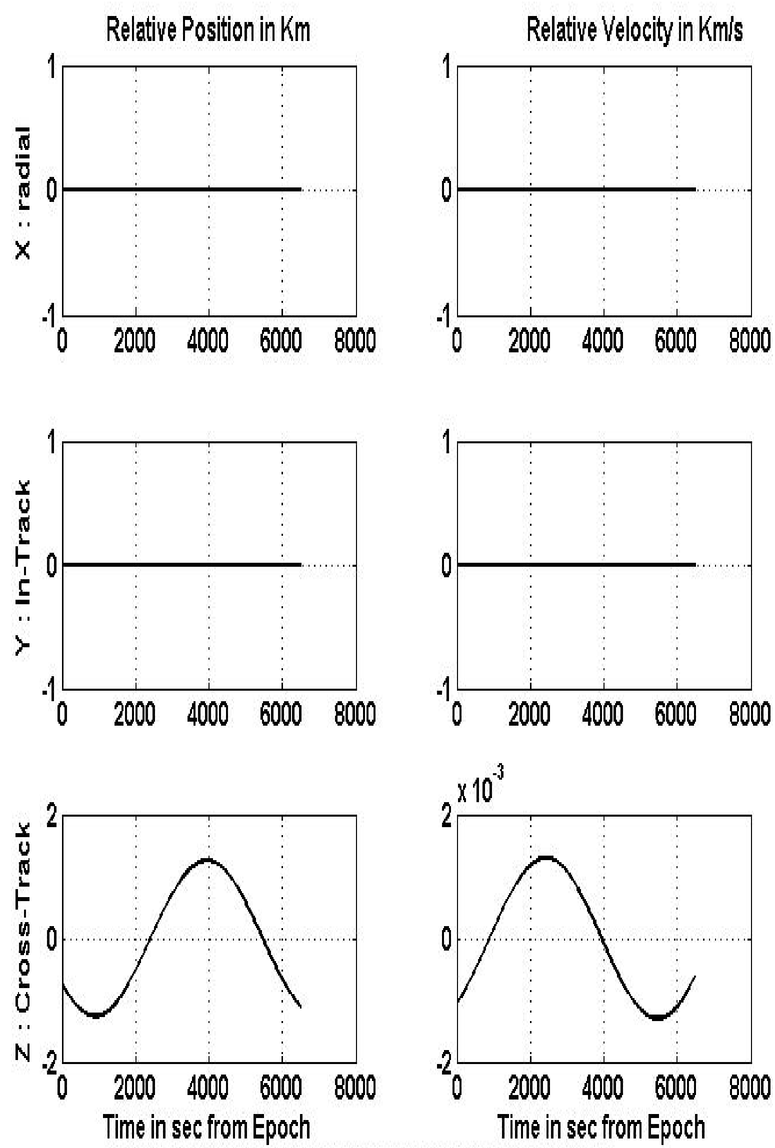


Effect of 1.0 Deg separation in Argument of Perigee at Epoch

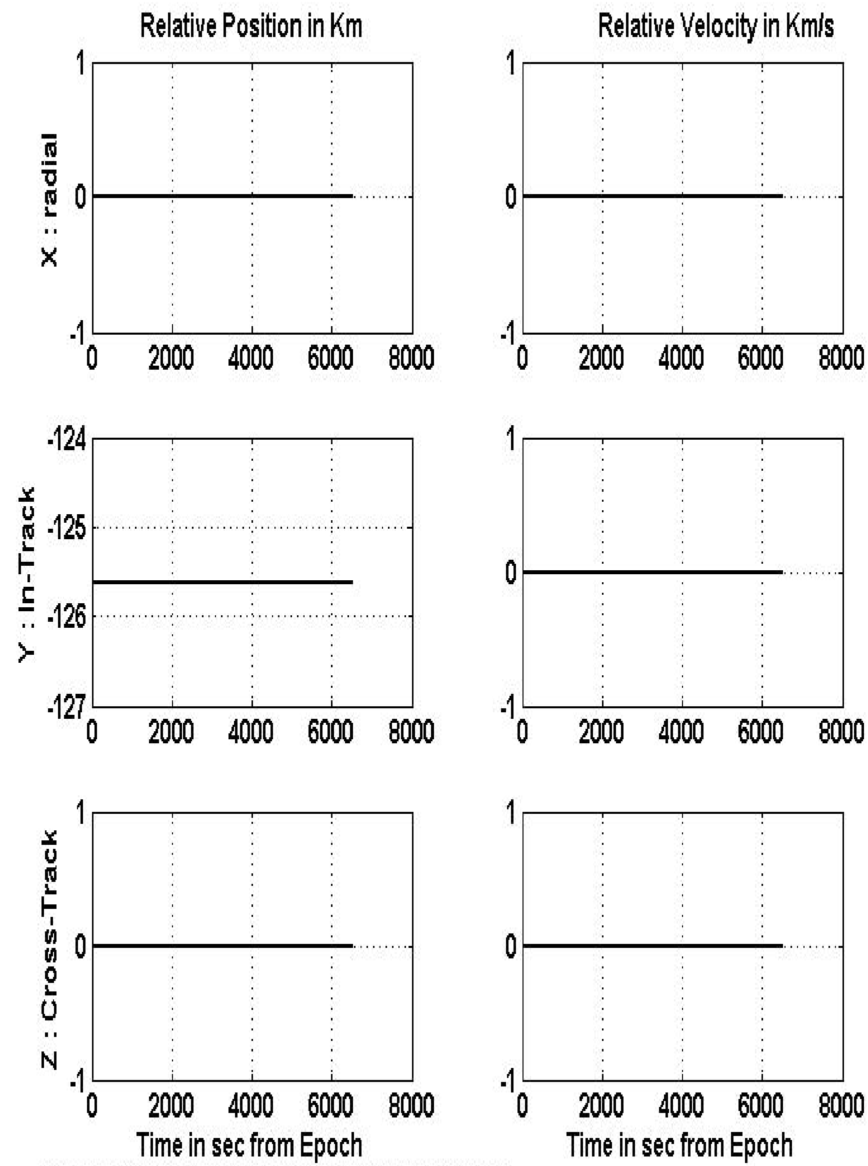


Effect of 0.01 Deg separation in Right Ascension at Epoch

Effect of Initial Separation in ‘i’ and ‘M (or θ)’ on Open Loop Position and Velocity



Effect of 0.01 Deg separation in Orbit Inclination at Epoch



Effect of 1.0 Deg separation in Anomaly at Epoch

Closed Loop Control Performance

Dynamics Parameters

Parameter	Value
S/C Mass	80.0 Kg
Maximum Thrust	0.2 N
Orbit Altitude	817.0 Km
Inclination	98.77 °
Right Ascension	193.0 °
Maximum Wheel Momentum	0.36 NmS @ 8000 RPM
Maximum Wheel Torque	0.018 Nm
S/C Inertia about Yaw, Roll and Pitch	
Leader	10, 9, 9.5 Kg-m^2
Follower	11, 8, 9 Kg-m^2
Star Tracker Update time for Gyro	128 ms
GPS update period for IRAP based position	10.0 s

Simulation Scenario

- Leader's Orbital Parameters a, e, i,Ω,ω, and M at Epoch (start) are (7198 Km, 0.0005176, 98.6868 °, 191.80 °, 323.397 ° and 36.605 °)

- Initially Leader is Nadir Pointing and Follower is away from Leader in position by [-68 m -214 m -1500m] in radial, along track and cross track direction. This corresponds to relative error of 100 m less in 'a' , 3 % less in 'e' and 0.01° less in 'i'

- From this the follower is reconfigured to relative position of [0.0, 100 m, -200 m] position and by control and maintained (i.e., zero relative velocity).

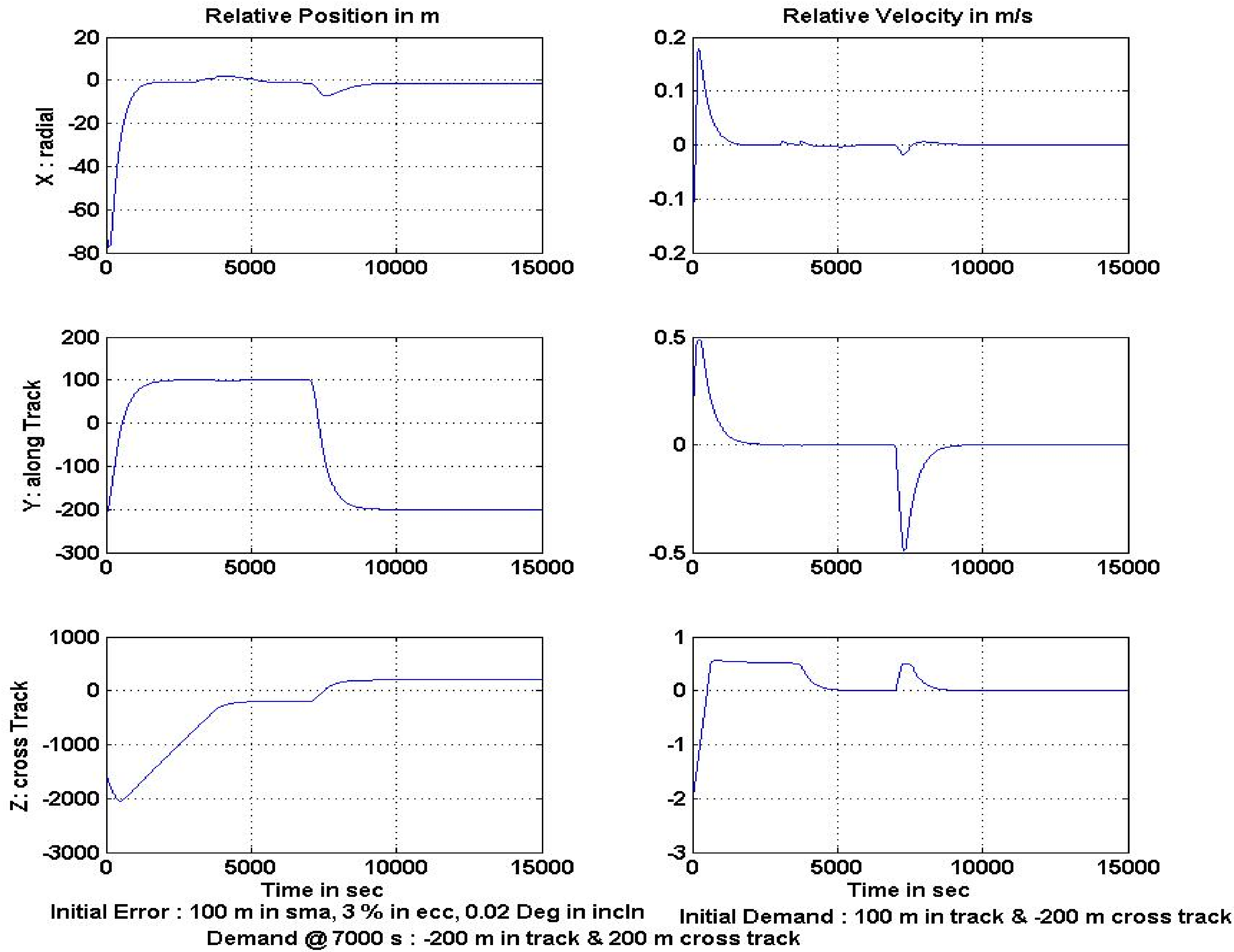
Initial Conditions & Mission Requirements (Typical simulation case)

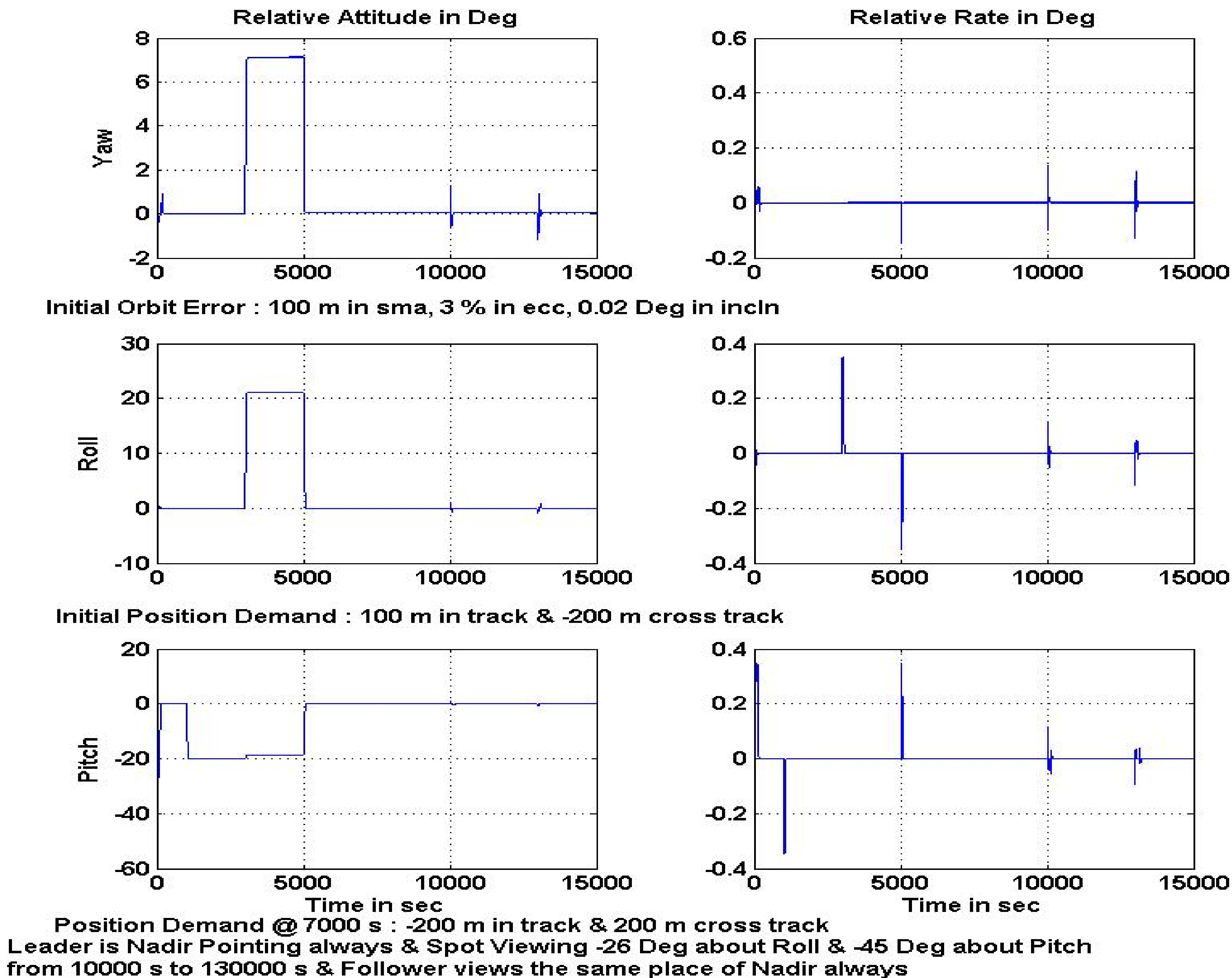
- Around 7000s, the follower is reconfigured to [0.0, -200m, 200m] relative position and zero relative velocity.

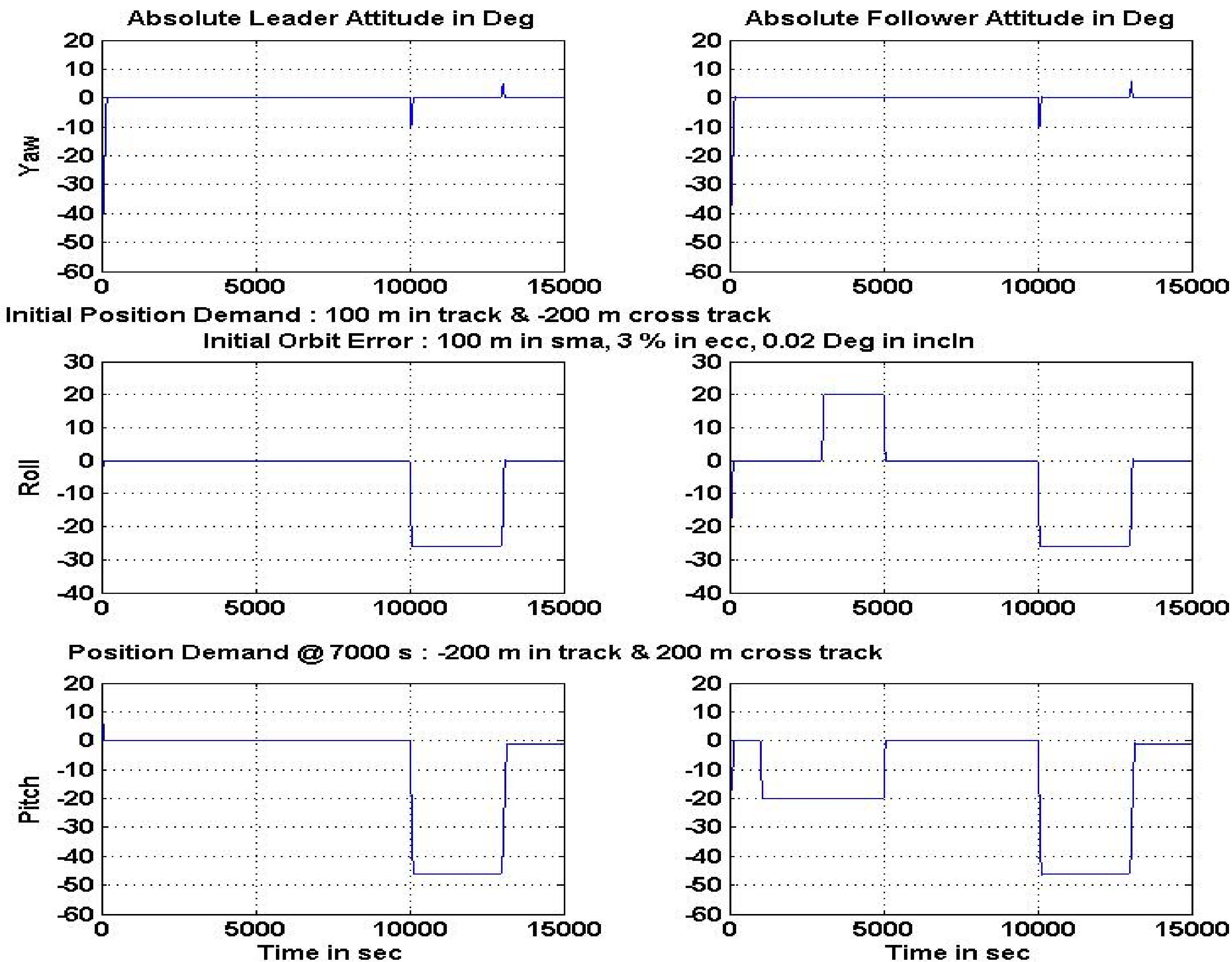
- From 10000 to 13000s, Leader is viewing off-track spot by rotating -26 ° about Roll (side view) and -45 ° about Pitch (Back View).

- By control action, the follower also points the same spot as that of Leader

- Thus we will get stereo view of the spot by Leader and Follower at different look angles


Relative Position in m
Relative Velocity in m/s
X : radial
Y : along Track
Z: cross Track
20
0
-20
-40
-60
-80
0.2
0.1
-0.1
-0.2
200
100
-100
-200
-300
0.5
-0.5
1000
-1000
-2000
-3000
1
-1
-2
-3
5000
10000
15000
Time in sec
Time in sec
Initial Error : 100 m in sma, 3 % in ecc, 0.02 Deg in incln
Initial Demand : 100 m in track & -200 m cross track
Demand @ 7000 s : -200 m in track & 200 m cross track


Relative Attitude in Deg
Relative Rate in Deg
Yaw
8
6
4
2
0
-2
0.6
0.4
0.2
0
-0.2
0
5000
10000
15000
Initial Orbit Error : 100 m in sma, 3 % in ecc, 0.02 Deg in incln
Roll
30
20
10
0
-10
0.4
0.2
0
-0.2
-0.4
Initial Position Demand : 100 m in track & -200 m cross track
Pitch
20
0
-20
-40
-60
0.4
0.2
0
-0.2
-0.4
Time in sec
Time in sec
Position Demand @ 7000 s : -200 m in track & 200 m cross track
Leader is Nadir Pointing always & Spot Viewing -26 Deg about Roll & -45 Deg about Pitch
from 10000 s to 130000 s & Follower views the same place of Nadir always



Leader is Nadir Pointing always & Spot Viewing -26 Deg about Roll & -45 Deg about Pitch
from 10000 s to 130000 s & Follower views the same place of Nadir always

Conclusions

- Effects of Initial Orbit Element Differences on Position and Velocity is studied

- A closed loop control scheme for relative position and attitude control has been demonstrated using LQR based controller

- It is observed that Stereo view of Spots (SPOT IMAGES) is possible with Leader-Follower relative Control

- Modeling of Navigation System Errors and Tuning of Kalman Filters form the future part of the work

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4. S D’Amico, O Montenbruck, C Arbinger and H Fiedler, “ Formation Flying Concept for Close Remote Sensing Satellites”, AAS-05-156, 15th AAS/AIAA Space Flight Mechanics Conference, Colorado, January, 2005
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Thank You