COORDINATED 6-DOF CONTROL OF DUAL SPACECRAFT FORMATION

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Abstract: *This paper provides an innovative dynamics and control algorithm for a dual-microsatellite formation flying mission with the application focusing on high resolution stereo imaging. The system concept is based on a six Degree of Freedom model where each follower generates the position and attitude references in real-time, based on the respective relative position and relative attitude motion between them and the leader, which also ensures that all the spacecrafts of the cluster are pointing at the target during data collection and even during formation reconfiguration. The system uses an Inertial Reference Accelerometer Package (IRAP) based referencing with update by GPS and Star Tracker for the position and attitude information loops respectively. The desired system parameters are specified in terms of orbital elements and attitude quaternion.*

Keywords: *Relative Navigation, Coupled Dynamics and Formation Keeping.*

1. Introduction

Advancement in the area of high resolution imaging demands several new challenges in satellite based cartography and stereo image generation. Invariably the high resolution data quality demands increase in size of the imaging instruments. Synchronized control of relative position and relative attitude in spacecraft formations has received increased attention over the last years. Small and micro spacecrafts in formation flying, based on the multi-agent control system, provide an approach towards meeting this mission goal. Indeed, there are many potential advantages of multi-satellite formation flying over a single satellite, in terms of greater flexibility, adaptability and improved performance. Maintaining a cluster of satellites in formation over their entire lifetime of a mission is a significantly more challenging control problem. It is required to meet stringent relative position in the presence of perturbing forces. This requires active control of position also. As thrust direction is a function of attitude, the control problem becomes a function of both position and attitude. This coupled control problem in turn also helps to improve the image accuracy as both orbit and attitude dispersions are minimized by the active control action.

The advantages of using spacecraft formations come at a cost of increased complexity and technological challenges. Formation flying introduces a control problem with strict and time-varying boundaries on spacecraft reference trajectories. Further collision among spacecraft should be avoided at all costs. The rise of spacecraft formation flying as a new technology has resulted in new areas of research, and the concept requires detailed knowledge and tight control of relative distances, velocities and orientations for participating spacecraft. A challenge for tight spacecraft formation flying lies in the coordination of the spacecraft motions relative to each other, to avoid inter-satellite interference and collisions and to achieve the specified mission goals, while minimizing the required control efforts. In addition, tight spacecraft formations are sensitive to perturbations due to external disturbances caused by atmospheric and solar drag, and variations in the gravity field of the Earth, and a solution to the control problem must be able to suppress the effect of these perturbations.

For relative position control, Kristiansen and Nicklasson [1] has compared several control techniques like Proportional + Derivative, Lyapunov, sliding in terms of position surface, velocity surface and backstepping. He has observed that Lyapunov method has slight overshoot in terms of error, while error achieved with PD control is lowest. Yang et al [2] has studied the precise navigation for GRACE formation mission using accelerometer together with K-Band pseudorange and IGS GPS ephemeris. They have concluded that position accuracy of 20 cm and velocity of the order of few mm/s are achievable. He et al [3], D'Amico and Montenbruck [4] brings out impulsive control approach schemes for relative eccentricity / inclination vector alignment using Gauss Perturbation Equation to achieve high efficiency / low fuel formation. He and Han [5], Jiang et al [6], Han and Yin [7] uses Relative Orbit Elements (ROE) based control laws to get the required inplane and out-of plane maneuvers. Le et al [8] performs a linear transformation from Hill frame to 2D frame in along track thus converting a coplanar spacecraft to equiangular one with wheels. Vaddi et al [9] discusses about the nonlinearities and perturbations of formation dynamics. Ardanens and D'Amico [10] discusses about the autonomous formation control of TanDem-X missions.

Zou and Kumar [11] works out terminal Sliding Mode Control for distributed attitude coordination among the formation members and confirms the stability between reaching phase and sliding phase by Lyapunov approach. Min et al [12] uses the adaptive control approach for coordination control. Vasconcelos et al [13] provide a theoretical work on GPS/IMU based Sensor fusion for estimation of relative navigation. Shan [14] and Chung et al [15] address the synchronization problem among the formation members. VanDyke and Hall [16] uses behavioral weightage based control laws for decentralized attitude control of spacecraft formation. Florio and D'Amico and Montenbruck [17] address the issues of collision avoidance in close remote sensing formations used for Interferometry. Florio and D'Amico [18] discusses on autonomous orbit control of remote sensing satellites to get separation of about 250 m. How and Tillerson [19] analyzes the impact of sensor noise and unmodeled measurement errors on formation control. Long and Hall [20] models the spacecrafts as individual wheels and derives attitude control law as a function of momentum and attitude. Gao et al [21], Xing et al [22] addresses the kinematics model associated with relative attitude control.

In case of single spacecraft, only attitude control is performed autonomously while the orbit control will be performed from ground. This is because attitude requirements are stringent and of the order of arc-sec level for imaging and the orbit control is required for ground track maintenance which is relaxed to the extent of 1 Km. In spacecraft constellations or proximity missions, only relative orbit control is required without stringent requirements on attitude control. But in formation flying spacecraft missions intended for Stereo imaging or SAR interferometry, it is required to have autonomous control of both relative position and relative attitude. This is because overall mission objectives can be met only when relative attitude and relative position are maintained collectively in the formation group.

Linear and nonlinear formation dynamic models have been developed in recent years for formation maintenance. Newer control concepts have been evolved to achieve the desired performance. Some systems use continuous control scheme while others use single impulse or multiple impulses to maintain the relative position. Continuous control results in large fuel consumption while the impulse control results in attitude transients which is a concern for high precision imaging. Stereoscopic imaging requires formation baseline of about 1 or 2 Km while the interferometry usage requires baseline of the order of 100 m. As a general guideline, to meet the baseline requirement of ‘x’, the control accuracy should be better than x/10 and the navigation or sensing accuracy should be better than x/100.

Ultra-close (baseline less than 100m) spacecrafts formation flying has attracted much attention and has been used recently since it can be implemented as interferometry observe missions, or, as a test programme for real autonomous rendezvous, docking and proximity operations. The larger the error, the higher is the fuel usage, and shorter the mission life. At the same time, precise relative attitude knowledge also provides a key factor to maintain mission requirements. In our previous paper [23], we have provided a basic concept of integrated formation control without modeling of sensors and actuators

In this paper, an innovative dynamics and control algorithm is developed for a dual-microsatellite formation flying mission with application focusing on high resolution stereo imaging. The system concept is based on a 6 Degree of Freedom model where each follower generates the attitude references in real-time, based on relative position and translational motion between the leader and its followers, which also ensures that the spacecraft are pointing at the target during formation reconfiguration. For the orbit control loop, the desired parameters are specified in terms of orbital elements and GPS measurements which measure the Cartesian coordinates. Desired attitude reference is derived on-board from the specified Earth view co-ordinates. Star tracker provides the attitude measurements.

2. Mathematical Modeling

Fig 1 shows the definition of spacecraft orbital elements in inertial frame.

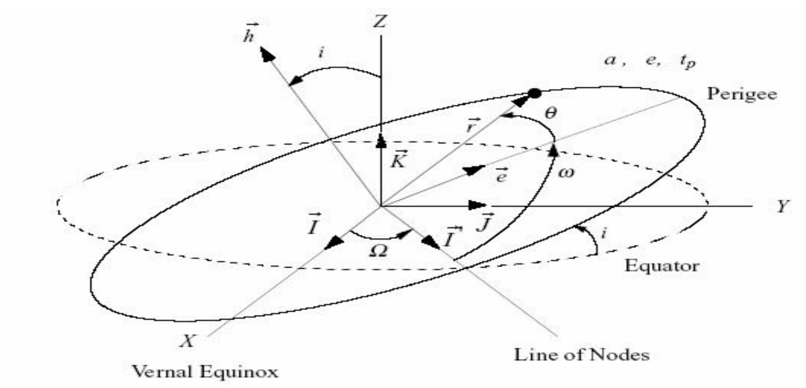

Figure 1. Schematic of Orbital Elements Definition

Relative Position Dynamics

The Keplerian elements of the leader and follower are used to define relative eccentricity vector Δe and relative inclination vector Δi as

$$\Delta e = e_L \begin{pmatrix} \cos\omega_L \\ \sin\omega_L \end{pmatrix} - e_F \begin{pmatrix} \cos\omega_F \\ \sin\omega_F \end{pmatrix} \quad (1)$$

and

$$\Delta i = \begin{pmatrix} i_L - i_F \\ (\Omega_L - \Omega_L)\sin i_F \end{pmatrix} \quad (2)$$

(Here the subscripts $_L$ and $_F$ are used to indicate leader and follower spacecrafts respectively)

The set of relative orbit elements is defined as

$$\Delta\alpha = \begin{pmatrix} \Delta a \\ a_F \Delta e_X \\ a_F \Delta e_Y \\ a_F \Delta i_X \\ a_F \Delta i_Y \\ a_F \Delta u \end{pmatrix} = \begin{pmatrix} a_L - a_F \\ a_F(e_L \cos\omega_L - e_F \cos\omega_F) \\ a_F(e_L \sin\omega_L - e_F \sin\omega_F) \\ a_F(i_L - i_F) \\ a_F(\Omega_L - \Omega_F)\sin i_F \\ a_F(u_L - u_F) \end{pmatrix} \quad (3)$$

The relation between unperturbed HCW equations and the relative orbit elements are shown in [4] as

$$\begin{bmatrix} \Delta r_x \\ \Delta r_y \\ \Delta r_z \\ \Delta v_x \\ \Delta v_y \\ \Delta v_z \end{bmatrix} = \begin{bmatrix} 1 & -\cos u & -\sin u & 0 & 0 & 0 \\ -1.5(u - u_0) & 2\sin u & -2\cos u & 0 & \cot i & 1 \\ 0 & 0 & 0 & \sin u & \cos u & 0 \\ 0 & n\sin u & -n\cos u & 0 & 0 & 0 \\ -1.5n & 2n\cos u & 2n\sin u & 0 & 0 & 0 \\ 0 & 0 & 0 & n\cos u & n\sin u & 0 \end{bmatrix} \Delta\alpha \quad (4)$$

where u and u_0 are the mean argument of latitude at any time t and epoch time t_0 respectively and 'n' is the mean orbital rate in rad/s.

Relative Attitude Dynamics

Relative attitude invove two parts namely the relative attitude kinematics and relative angular rate dynamics.

Relative attitude Qauternion of the follower spacecraft with respect to the leader is given by

$$q = q_L^* \otimes q_F \quad (5)$$

where, $\mathbf{q_L}$ and q_F are the inertial attitude of the leader and follower spacecrafts, respectively. The symbol $\otimes$ refers to Quaternion multiplciation and * indicates the Quaternion conjugate.

Relative attitude kinematics is given by

$$\dot{q} = \frac{1}{2}\left(q \otimes \omega\right) \tag{6}$$

where, ω represents the relative angular velocity of the follower with respect to the leader.

$$\omega = \omega^{F}_{L,F} = \omega^{F}_{I,F} + \omega^{F}_{L,I} = \omega^{F}_{I,F} - \omega^{F}_{I,L}$$

or

$$\omega = \omega^{F}_{L,F} = \omega^{F}_{I,F} - T\left(q_{LF}\right)^{*}\omega^{L}_{I,L} \tag{7}$$

where I, L, F, respectively refers to the Inertial Frame, leader's Orbit Frame and follower's orbit frame and $T(q_{LF})$ represents the attitude rotation from leader to the follower frame.

The notation $\omega^{C}_{A,B}$ indicates the angular velocity of frame 'B' with respect to frame 'A' as represented in frame 'C'.

Relative angular rate dynamics is derived using (8) along with the rate dynamics of leader and follower spacecrafts which are obtained by the Euler moment equation,

$$J_s\dot{\omega} = -\omega \times J\omega - \omega \times H_w + T_c + T_d \tag{8}$$

where, J is the moment of inertia of the spacecraft (s = L or F), H_w is the angular momentum developed by wheel about the spacecraft principal axes, T_c and T_d, respectively are the control torque and disturbance torque.

3. Multi sensor Data Fusion Scheme

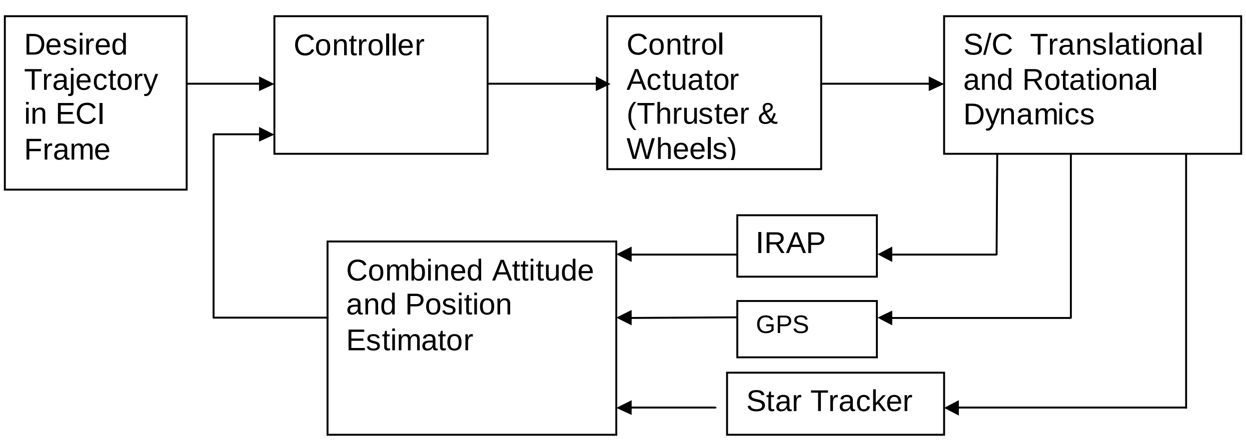

Figure 2 : Block Diagram of Satellite Formation Control Loop (Coupled Position and Attitude)

Figure 2 shows the block diagram of the control scheme used in this work. Desired reference trajectory is specified in terms of the relative orbit elements. GPS + IRAP +

Star Tracker forms the feedback control path. An IRAP unit comprises of an accelerometer and gyro

Attitude Loop

Gyro, together with the star tracker forms the attitude information source for the attitude control loop. This provides the instantaneous attitude of the spacecraft with respect to the Inertial Frame which is used along with the gyro rate for the leader. For the follower spacecraft control, the relative attitude and rate of the follower with respect to the leader's orbit reference frame as represented in the follower frame is obtained as described earlier.

Position Loop

Accelerometer measures the acceleration in body frame which is represented in inertial frame using transformation matrix obtained from the instantaneous body to inertial attitude quaternion (which is the conjugate of the feedback attitude used for attitude loop). This transformed acceleration is double integrated and used to propagate the position information between absolute GPS updates. The processed Cartesian data are transformed to orbital elements and fed to the controller.

LQR control concept is used for control of both position loop and attitude loop as the same can achieve the desired goal with minimum control effort compared to conventional PD controller.

Absolute position and attitude control is used for the leader spacecraft and the relative control for the follower spacecraft. Reference attitude for the leader spacecraft is specified based on the required imaging target locations and view angle required and the orbit control reference is the required position and velocity to maintain the specified ground track. GPS, together with the star tracker, provides the current navigation information of the spacecraft. These information sources are optimally fused with that of IRAP output by a Kalman filter. Relative Translational estimates are mapped to mean elements and fed as control input for orbit while the attitude and rate estimates are used for the rotational part of the control. The dynamics uses the full fledged model to closely match the actual environment and control and estimation and controller are based on linear models.

Controller design

Linear Quadratic Controller is used for both position and attitude control loops. This is based on optimal control theory wherein a set of differential equations describe the trajectory to optimize a specified cost functional. For the position loop, wherein the thruster is the main actuator, fuel optimization is the main cost functional. For the attitude loop, whose main role is in maintaining thrust direction and reorientations to favorable geometry for imaging, the maneuver time is the main cost functional.

The cost functional for the LQR controller is given by

$$J = \frac{1}{2}\int\left[x^T Q(t)x + u^T R(t)u\right]dt + \frac{1}{2}x^T(t_f)Q_t x(t_f) \quad (9)$$

Where, Q(t) is a positive semidefinite state weighting matrix, R(t) is a positive definite control weighting matrix and Q_t is positive definite terminal – weighting matrix. The objective is to find the feedback gain K which minimizes the cost functional J.

The optimum feedback gain is given by

$$K = R^{-1}B^T P(t) \quad (10)$$

where, the matrix P is computed by solving the continuous time Riccatti equation

$$\dot{P}(t) + A^T P(t) + P(t)A - P(t)BR^{-1}B^T P(t) + Q = 0 \quad (11)$$

with the boundary condition $P(t_f)=Q_t$

4. Simulation and Analysis

Two microsatellites of Mass 80 Kg and moment of inertia approximately 10 Kg-m^2 are considered in a leader-follower formation. A thruster of 0.2 N and four micro reaction wheels of 0.36 NmS angular momentum at 8000 RPM and 0.018 Nm torque capacities are used as control actuator for the control loops.

The simulation parameters and initial conditions are summarized in table 1 below.

Table 1 : Simulation Parameters

Parameter	Value
S/C Mass	80.0 Kg
Maximum Thrust	0.2 N
Orbit Altitude	817.0 Km
Maximum Wheel Momentum	0.36 NmS @ 8000 RPM
Maximum Wheel Torque	0.018 Nm
S/C Inertia about Yaw, Roll and Pitch **leader** **follower**	 10, 9, 9.5 Kg-m^2 11, 8, 9 Kg-m^2
Star Tracker Update time for Gyro	128 ms
GPS update period for IRAP based position	10.0 s
Mean Orbital Elements of Leader at Epoch or start time (a,e,i,Ω,ω,M)	(7198 Km, 0.0005176, 98.6868 °, 191.80 °, 323.397 ° and 36.605 °)

Fig 3 shows the effect of difference in individual orbit elements at epoch on the relative position and velocity without any control action (open loop). Offset in initial semi major axis produces offset in radial position and drift in along track position. Error in eccentricity as well as argument of perigee (both are equivalent as eccentricity vector is directed towards perigee) produce in-plane orbit frequency component with phase difference between both effects. Similarly, error in right ascension and orbit inclination cause cross track periodic component with phase difference between both effects. Error in Epoch Mean Anomaly produce constant offset in along track direction.

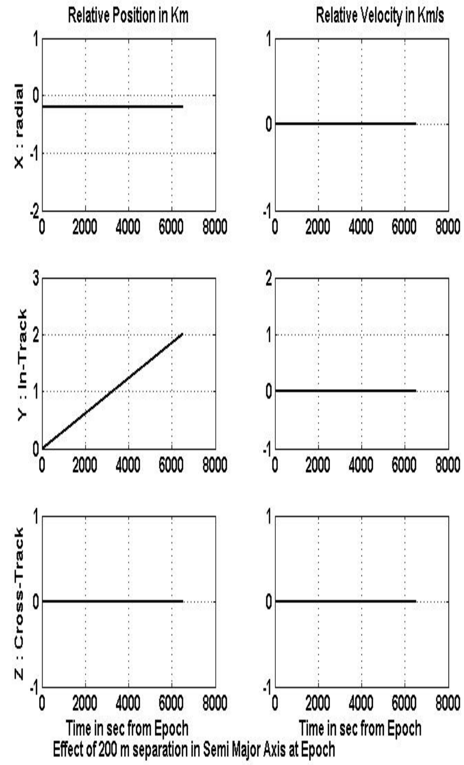

Fig 3(a)

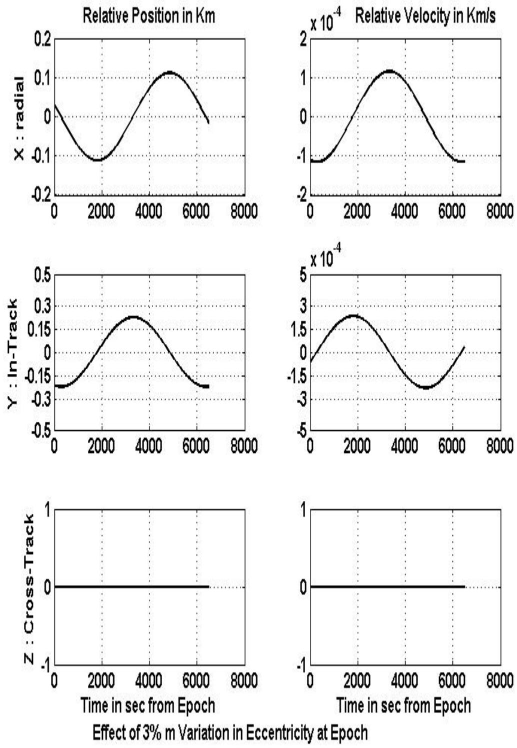

Fig 3(b)

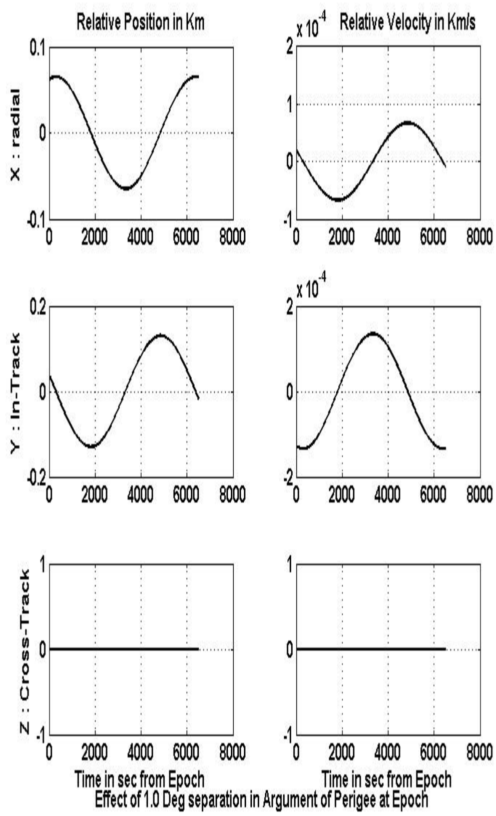

Fig 3(c)

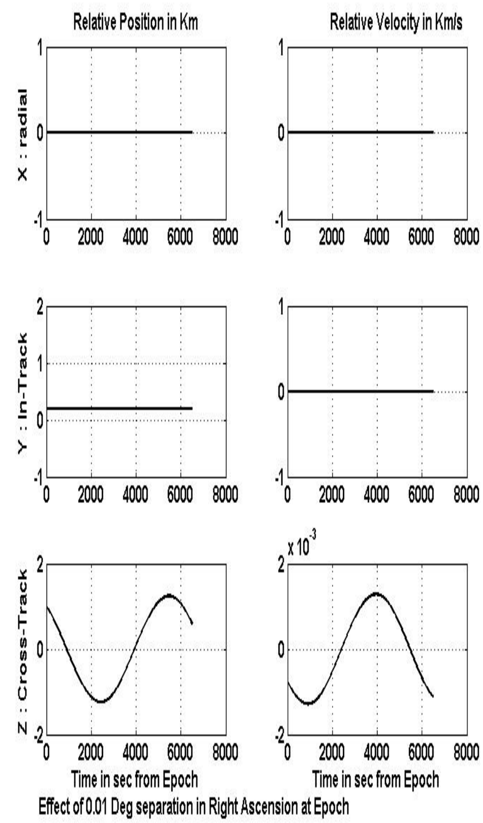

Fig 3(d)

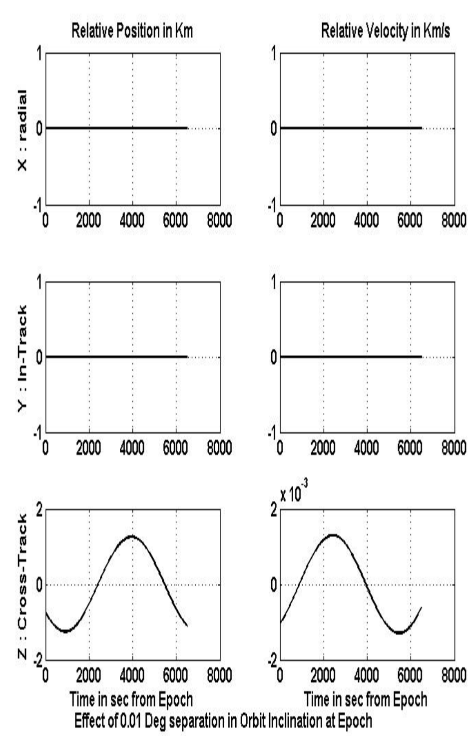

Fig 3(e)

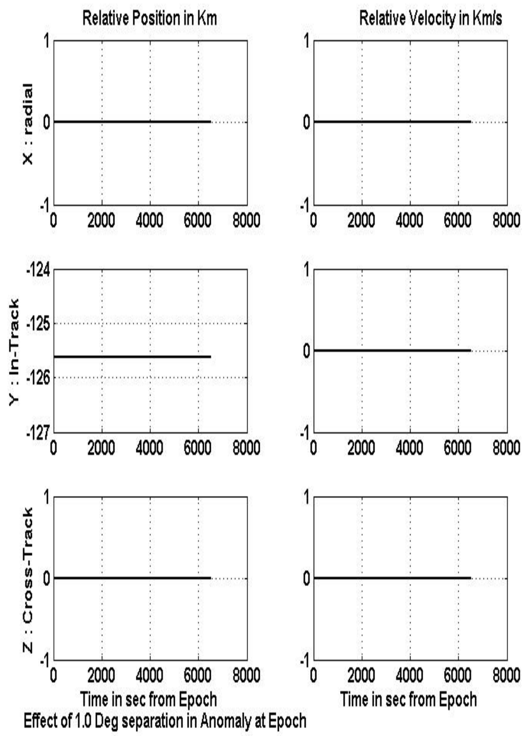

Fig 3(f)

Figure 3 : Effect of Initial Orbit Element Differences on Relative Position and Velocity

Closed Loop Control

Figure 4 shows the closed loop response of position and velocity of the follower spacecraft with respect to the leader by using the LQR controller. Perturbations due to J_2 and full-fledged non-linearities have been included in the plant dynamics. At epoch (start of the simulation), the follower spacecraft location with respect to the leader is 100 m less in semi major axis, 3 % less in eccentricity and 0.01° less in orbit inclination. This translates to [-68 m -214 m -1500m] error in relative position and [-.11 m/s 0.09 m/s and -2m/s] error in relative velocity. As seen in fig 4, from this the formation is controlled to the required condition of 100m in track and -200 m in cross track positions with zero relative velocities and zero radial distance which is required for imaging. At around 7000 s, the formation is reconfigured to -200m in track and 200 m cross track conditions.

Figure 5 shows the relative rotational (attitude and rate) response of the follower spacecraft with respect to the leader orbit as represented in follower's orbit frame. The follower is made to view the same image point of the leader. As attitude loop is fast compared to the position loop, during the convergence of relative position, attitude biases are seen to maintain the instantaneous look direction. Between 10000 sec to 13000 sec, the leader is off-nadir viewing (rotated in along and / cross direction) and it is seen that the follower is also following the same area with a short transient.

Figure 6 shows the absolute attitude of the leader and the follower with respect to their corresponding orbit reference frame. This shows that leader is always nadir pointing except imaging from 10000 to 13000s while the follower is off-nadir in its orbit frame initially and also during the convergence of relative position as it is required to maintain relative attitude with respect to leader.

From the simulations it is seen that though initial orbit error corrections took more time (due to low thrust), in between small reconfiguration of the formation in both translation and rotation are faster.

5. Conclusion

A closed loop control scheme for relative position and attitude control has been demonstrated using LQR based controller. The error dynamics of the measurement sensors have not been included in the analysis. Tuning of the Kalman Filter and statistical analysis of the measurement error sources will be performed in next phase of this work.

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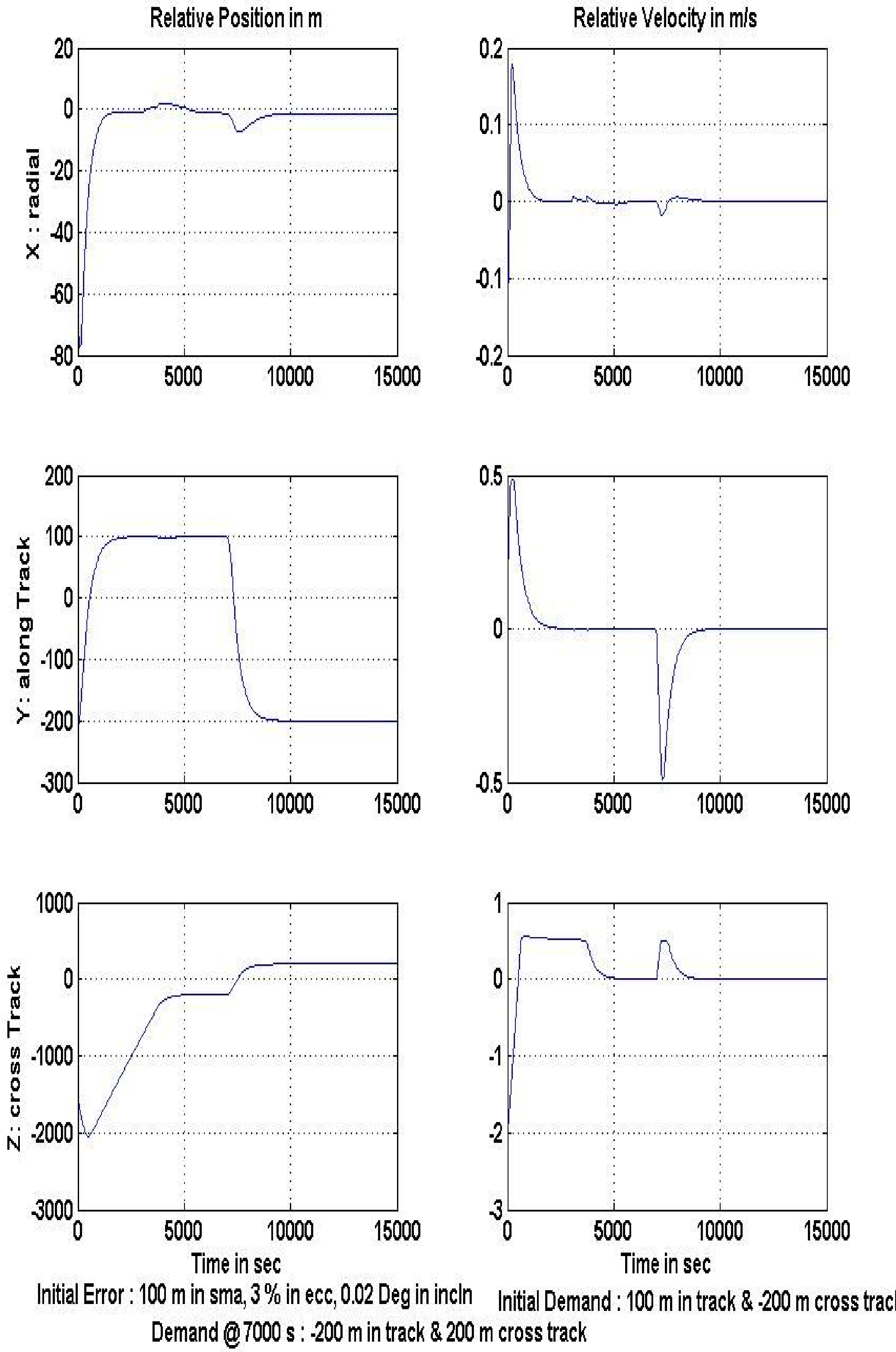

Figure 4 : Relative Position & Velocity Response during Closed Loop Control

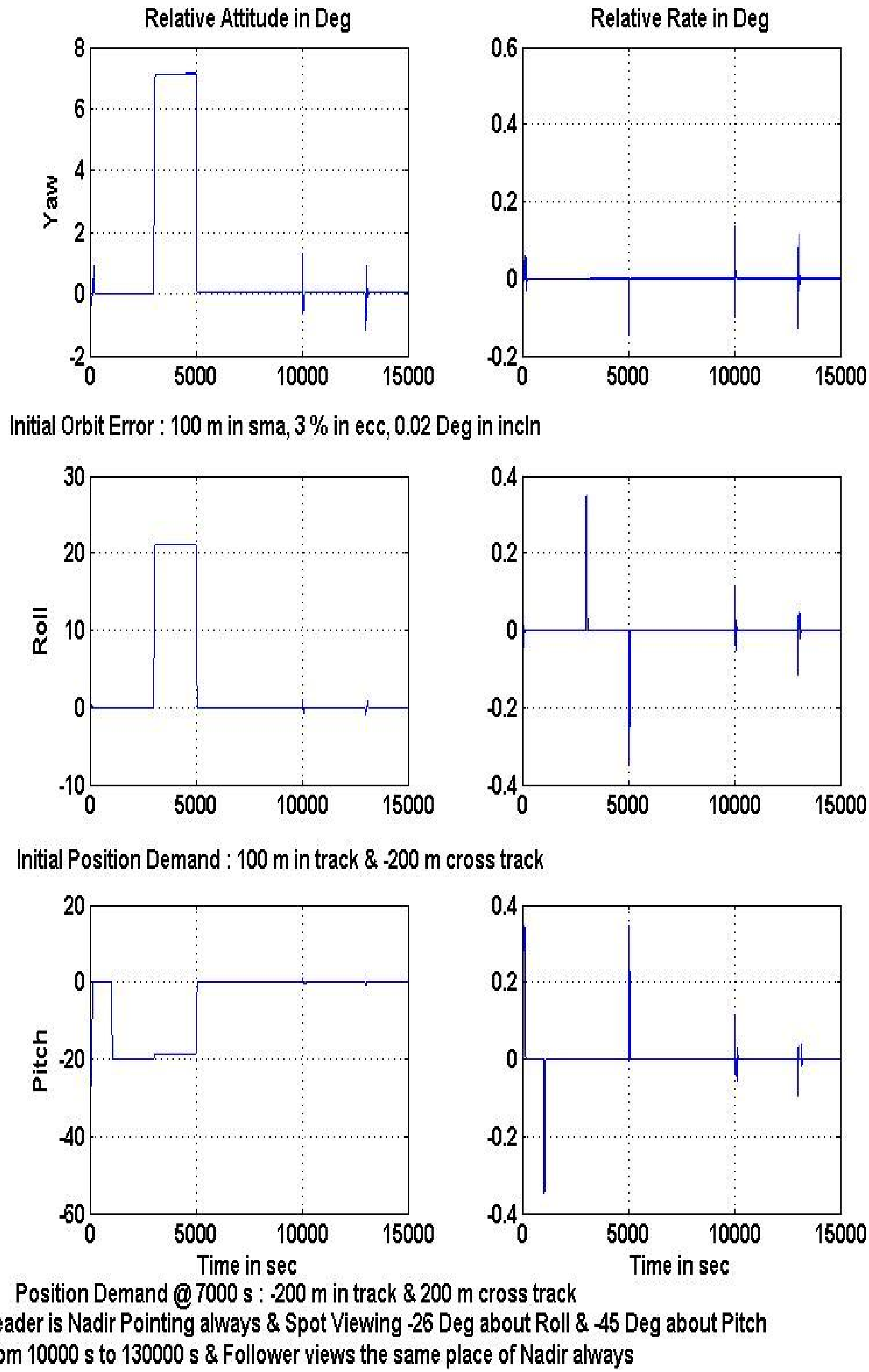

Figure 5 : Relative Attitude & Angular Rate Response during Closed Loop Control

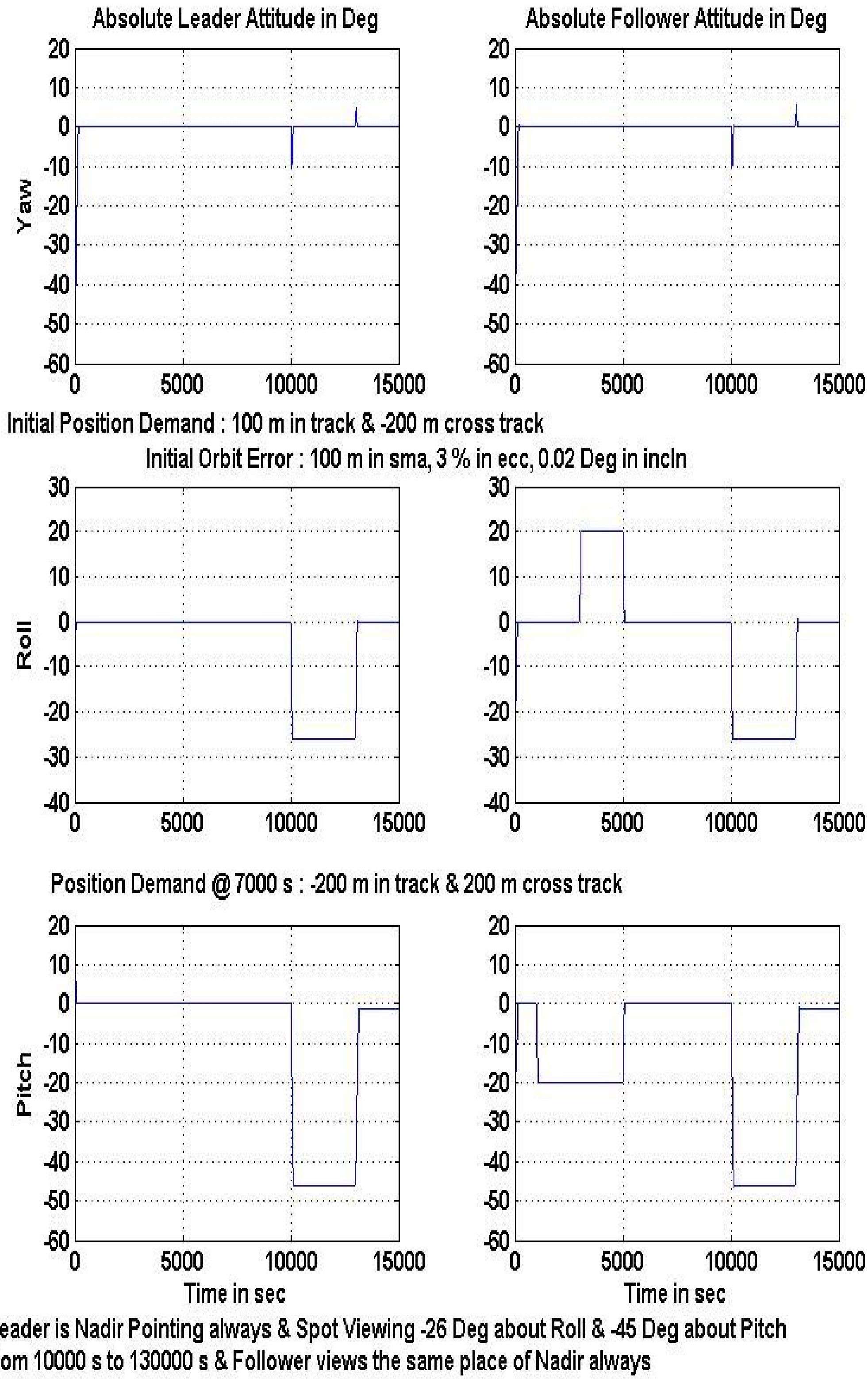

Figure 6 : Absolute attitude of leader and follower in their respective orbit frame

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