Comparison of different CDGPS solutions for on-the-fly integer ambiguity resolution in long baseline LEO formations

U. Tancredi, A. Renga, and M. Grassi

5th International Conference on Spacecraft Formation Flying Missions and Technologies
May 29-31, 2013, Munich, Germany
Problem Statement
Motivation and Objectives

Problem
- Precise real-time onboard relative positioning for long-baseline spacecraft formations in LEO (Low Earth Orbits)

Fields of Application
- Long baseline required for Bi-/Multistatic SAR (Synthetic Aperture Radar) applications
- Precise Real-time onboard relative positioning appealing for future distributed Earth Observation Systems based on multiple low-cost small platforms
  - *Onboard autonomous formation control and image generation*

Objective
- Selection of potential solutions towards real-time onboard CDGPS (Carrier-phase Differential GPS)
- Demonstration on GRACE (Gravity Recovery and Climate Experiment) data
CDGPS:
- Potential for cm-scale 3D accuracy based on the possibility to exploit the integer nature of DD (Double Difference) carrier-phase ambiguities

**CHALLENGE**

- **LEO operation**
  - Dynamic initialization of ambiguities

- **Long baseline**
  - Significant differential ionospheric delays

- **Real-time**
  - Substantial broadcast ephemeris errors

**SOLUTION**

- **On-The-Fly (OTF) Integer Ambiguities (IA) resolution**
- **Dual-frequency operation mandatory for OTF IA resolution**
- **Specific solutions for reliable IA estimates (e.g. EKF)**
Iono-delays appear in all pseudorange (PR) and carrier-phase (CP) DD observation equations.

- Four observations (2 PR + 2CP) per DD pair \( jk \)
- One iono-delay per each DD pair \( jk \)

Magnitude depends on the separation btw. S/C (baseline \( b \)) and on the ionosphere status

- For \( b \sim 200 \) km, \( \mathbf{I}^{jk} \) can be 0.5 m for mild iono.

Magnitude of delays vs. iono activity.

- 11-yr long solar cycle induces variations of DD delays of one order of magnitude
For **OTF ambiguity resolution**, several techniques exist for handling iono-delays.

### PROS
- **Cancellation by measurement combination**
  - No 1st order modeling error

- **Estimation of individual DD iono-delays**
  - Simple Modeling (e.g. Random Walk, etc.)

- **Approximation by Lear’s model**
  - Reduced number of unknowns

### CONS
- Reduced number of independent observations
- Increased number of unknowns
- Systematic errors

- **Individual estimation similar to cancellation** when time correlation is low
Rationale: Two-step approach

• On-The-Fly (OTF) Ambiguity Resolution
  • Computes IAs using as little computational effort as possible
  • Uses time correlation of geometry and cycle ambiguities (Kalman Filter)

• Real-Time Kinematic Relative Positioning
  • Uses only Carrier Phase measurements and computed IA
  • Achieves precise positioning irrespective of previous models errors

---

1 U. Tancredi, A. Renga, M. Grassi, Real-time relative positioning of spacecraft over long baselines, AIAA Journal of Guidance Control and Dynamics, Accepted for publ., 2013
EKF with Partial Closed/Loop Scheme

Extended Kalman Filter (EKF) used to compute the float solution

Least-squares AMbiguity Decorrelation Adjustment (LAMBDA) method (Teunissen 1995) used to estimate Integer Ambiguities

Partial C/L based on the validation of Wide-Lane (WL) ambiguities: robustness demonstrated for real-time onboard scenario

Single-epoch L1 ambiguities fixing by a secondary LAMBDA with no validation
Relative dynamics in ECEF described by a nonlinear **Keplerian model** corrected for **differential J2 effects**

\[
\ddot{b} = \frac{\mu}{r_c^3} \left[ C_1 (r_c) \cdot r_c + C_2 (r_c, b) \cdot (r_c + b) \right] + \Omega_E \times \Omega_E \times b + 2\Omega_E \times \dot{b} + w_b
\]

- **Analytical Jacobian available**
- **Trade-off for a computational load** adequate for real-time onboard implementation
- Implementation of the **absolute dynamics** of the chief satellite required

**Hybrid solution** selected for absolute dynamics
- The same model is used to propagate also the absolute chief satellite starting from the internal solution of the receiver
- **Chief satellite position not retained** after the propagation but computed at each time epoch
Solution 1: Approximation by Lear’s model

- Different VTEC for the two receivers
- Mapping function (Lear, 1987) models the ratio of the slant TEC to the vertical TEC as a factor of the elevation on the horizon of GPS Satellite Vehicles (SV) as measured by the receiver.

\[
m_{\text{rec}}^v = \frac{40.3}{f_1^2} \cdot \frac{2.037}{\sin^2(E_{\text{rec}}) + \sqrt{\sin^2(E_{\text{rec}})}} + 0.076
\]

**State & Measurement Vectors**

\[x \in \mathbb{R}^{(8+2p) \times 1}, \quad x = \left(b^T \quad \text{VTEC}^T \quad a_w^T \quad a_1^T\right)^T\]

\[y \in \mathbb{R}^{4p \times 1}, \quad y = \left(P_1^T \quad P_2^T \quad L_1^T \quad L_2^T\right)^T\]

**Observation Model**

\[
h(x) = \left(\begin{array}{c}
I_p \\
I_p \\
I_p \\
I_p \\
I_p \\
I_p \\
I_p \\
I_p \\
\end{array}\right) \rho_{CD}^j (b') + \left(\begin{array}{c}
I_p \\
I_p \\
\gamma^{-2}I_p \\
\gamma^{-2}I_p \\
-I_p \\
-I_p \\
0_p \\
0_p \\
\end{array}\right) \mathbf{I}_{CD}^j (\text{VTEC}) + \left(\begin{array}{c}
0_p \\
0_p \\
0_p \\
0_p \\
-\lambda_1I_p \\
-\lambda_1I_p \\
0_p \\
0_p \\
\end{array}\right) a_w + \left(\begin{array}{c}
0_p \\
0_p \\
0_p \\
0_p \\
\lambda_2I_p \\
\lambda_2I_p \\
0_p \\
0_p \\
\end{array}\right) a_1
\]
Extended Kalman Filter
Handling of DD ionospheric delays (2/2)

Solution 2: Cancellation by combinations of measurements

- GPS measurement combinations cancelling first-order ionospheric delays:
  - Ionospheric-free combinations, $P_{IF}, L_{IF}$
  - Group and Phase corrections (GRAPHIC), $G_1, G_2$
  - Melbourne-Wubbena combinations, MW
- Five types of combinations available, but only three are linearly independent

\[ x \in \mathbb{R}^{(6+2p) \times 1}, \quad y = \begin{pmatrix} b^T \quad a_w^T \quad a_i^T \end{pmatrix}^T \]

State & Measurement Vectors

\[ y \in \mathbb{R}^{4p \times 1}, \quad y = \begin{pmatrix} (L')^T \quad (G_1')^T \quad (MW')^T \end{pmatrix}^T \]

Observation Model
• Data available for Gravity Recovery and Climate Experiment (GRACE) mission, which has a Leader – Follower formation of two satellites, GRACE A and GRACE B

• Four DOYs from 2009-2011 selected for the performance assessment
  • DOYs differ in the ionospheric activity, from mild to intense
  • Baseline ~ 250 km, Orbit Altitude ~ 460 km
  • GPS measurements are available at 0.1 Hz rate

• “True” solution can be reconstructed using GRACE L1B data products
  • Geometry, using the International GNSS Service (IGS) and GRACE L1B data products
  • Ionospheric delays (van Barneveld, et al. 2009) and integer ambiguities (Tancredi, et al. 2011), using GRACE GPS measurements and true geometry
Positioning Performance on Flight Data

Selected Datasets

- DOYs selected for sampling increase in solar cycle (maximum in 2013)
  - DD Iono-delays increase of one order of magnitude, e.g. RMS from 5 to 40 cm.
  - VTEC in LEO ranges from 10 to 50 TECU
Solution with Lear’s model

- RMS baseline magnitude error < 5 cm
- IA Fail rate < 2%
- WL ambiguities always correct

No-ino solution

- RMS baseline magnitude error < 10 cm
- IA Fail rate < 10%
- WL ambiguities always correct
Positioning Performance on Flight Data
Oct. 2011 - Intense Ionosphere

Solution with Lear’s model
- RMS $\Delta|b|$ increases to 30 cm
- IA Fail rate up to 15%
- WL ambiguities still correct

No-iono solution
- RMS $\Delta|b|$ increases only slightly
- IA Fail rate still worse (20 %)
- WL ambiguities always correct
Solution with Lear’s model degrades proportionally to VTEC’s increase, both in baseline and IA.

No-ino ambiguity solution also degrades proportionally. Baseline is instead relatively insensitive to iono-activity. Lower correlation btw. baseline & L1 ambiguities.
Conclusion

Different approaches analyzed for ionospheric delay compensation in long baseline [~250 km] LEO formations

- Two approaches developed in detail and compared on actual flight data: Modeling vs. Deleting ionosphere

Results suggest ionospheric activity plays a major role in determining performance

- Modeling the ionosphere, the positioning performance proportionally degrades with iono-activity
- Positioning without iono-delays is less sensitive to ionospheric activity

Deleting ionosphere reduces observability of L1 ambiguities

- In mild iono-conditions, positioning is worse than iono-modeling
- In severe iono-conditions, wrong L1 ambiguities only marginally affect positioning