西北工業大學
NORTHWESTERN POLYTECHNICAL UNIVERSITY

REAL-TIME 6DOF TERMINAL GUIDANCE FOR AUTONOMOUS SPACECRAFT CAPTURE FREE FLOATING OBJECTS USING STATE DEPENDENT MODEL PREDICTIVE CONTROL

Author Peng LI

Joint Work with

Xiaokui YUE，Honghua DAI，Xianbin CHI

National Key Laboratory of Aerospace Flight Dynamics
Northwestern Polytechnical University

Outline

I、Introduction

- **Autonomous Rendezvous and Capture**: significant development has been made in this field, such as space mission:
  - OE : Orbital Express
  - SUMO: Spacecraft for Universal Modification of Orbits
  - DART: Demonstration for Autonomous Rendezvous Technology
  - XSS-11: Experimental Satellite System-11

- These programs demonstrate that there is a need for an effective **autonomous rendezvous optimal control algorithms** which drive the chaser spacecraft rendezvous with the target under practical constraints.

I、Introduction

- **Previous Research**:
  - **Lembeck** investigated the low-thrust rendezvous in circular orbit without constraint
  - **Guelman** investigated the optimal-fuel rendezvous in circular orbits with fixed terminal-approach direction
- **Limitations**: rendezvous problem was treated without considering realistic constraints, such as
  - Magnitude constraint of thruster activation
  - Collision avoidance
- **MILP**: applied in the optimal rendezvous trajectory planning under practical constraints
  - **Richards** introduced MILP method for finding fuel-optimal trajectory subject to collision avoidance and prevention of thruster plumes
  - **Breger** studied online generation of safe, fuel-optimized trajectories that guarantee collision avoidance when system anomaly happens

I、Introduction

- **Guess Pseudospectral Method**:
  - **Advantage:** better at dealing with smoothing optimal problem, receiving more attention recently
  - **Disadvantage:** difficult to get solution of the NLP, especially when there is a large number of nodes.

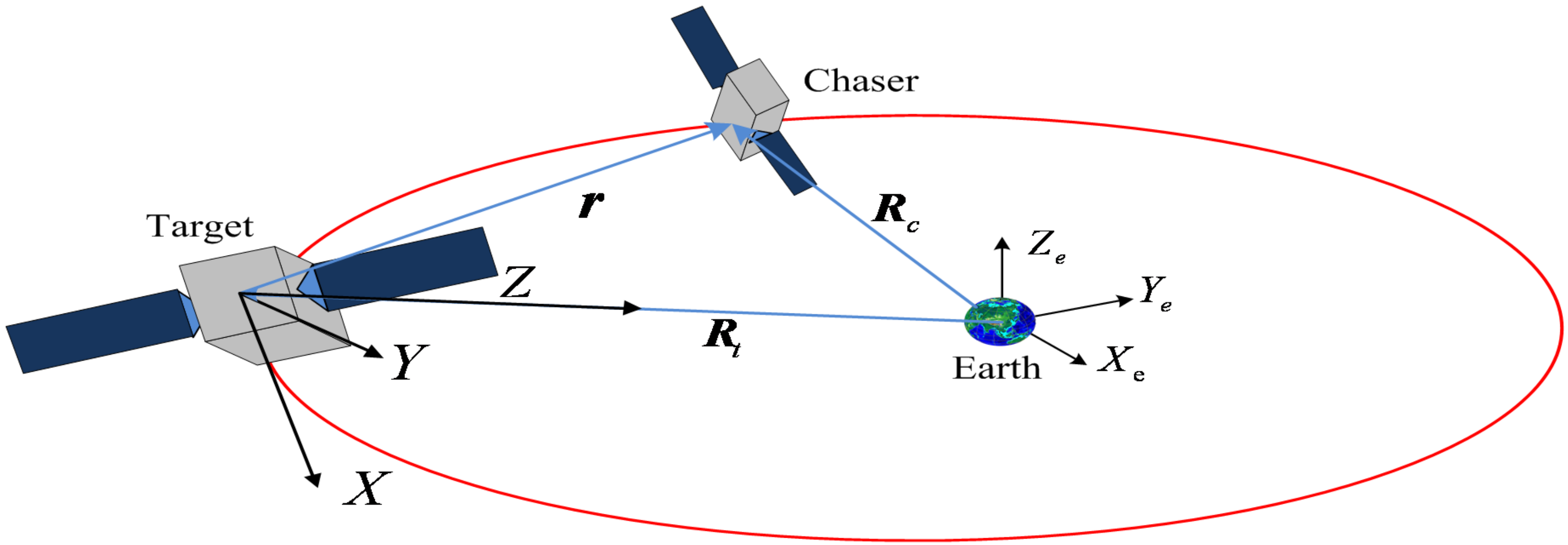
- **Artificial Potential Function**: Mclnnes developed a novel guidance and control methodology for path constrained proximity maneuvers of spacecraft at the ISS.

- **Model Predictive Control**:
  - Origins in the process industry, has recently been applied to trajectory planning of spacecraft
  - Possess some attractive characteristics, such as re-plans the optimal trajectory at each instant

I、Introduction

- **Why MPC is selected in the mission ARC ?**
  - ARC typically has many constraints: such as limited power system, collision avoidance and the dynamic characteristics of the target.
  - MPC re-plans the optimal trajectory at each instant based on the receding optimization strategy
- **However**， previous work mainly focused on 3 DOF trajectory planning, seldom studies the optimal attitude maneuver of the chaser spacecraft.
- **Reason**: Euler’s rotation dynamic equations are nonlinear model, hence, traditional linear optimization method are unsuitable any more.
- **Contribution of our work:** SDMPC algorithm is proposed which is better at dealing with not only linear translation model, but also nonlinear attitude model.

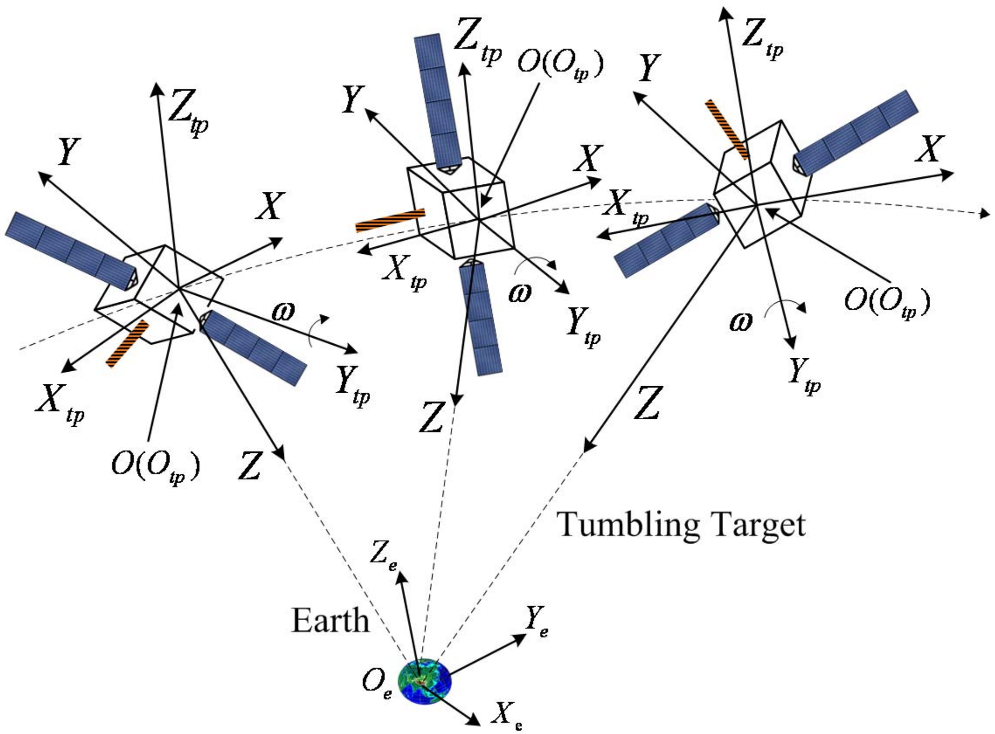
II.A、Relative Translation Dynamics



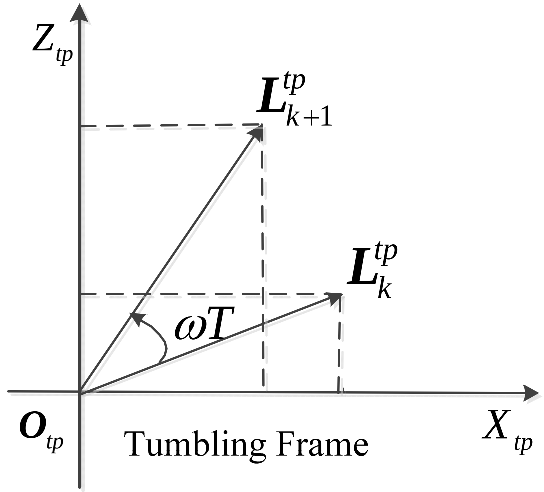
Hill–Clohessy–Wiltshire Model

$$\begin{cases} \ddot{x} - 2n\dot{z} = \dfrac{F_x}{m} \\ \ddot{y} + n^2 y = \dfrac{F_y}{m} \\ \ddot{z} + 2n\dot{x} - 3n^2 z = \dfrac{F_z}{m} \end{cases}$$

II.A、Relative Translation Dynamics



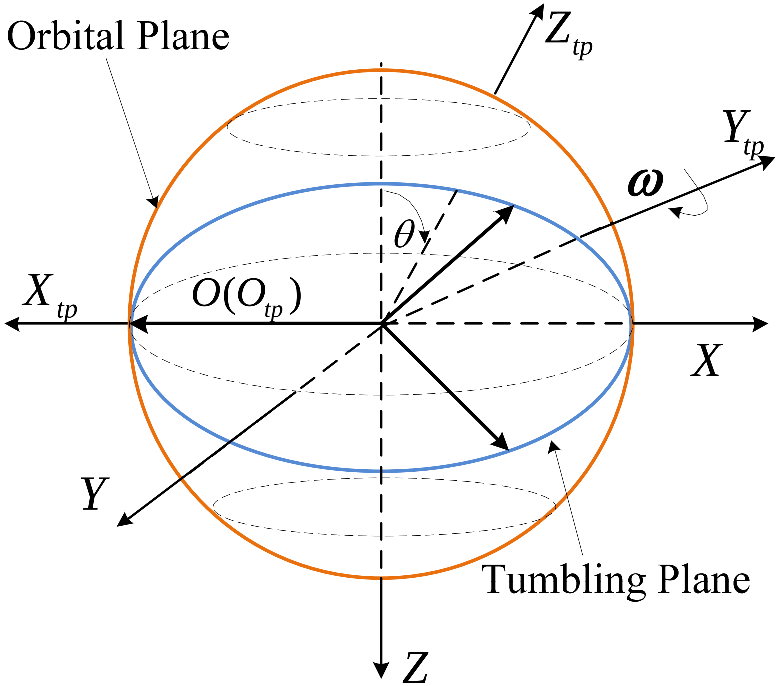
Relationships Between the Orbital Frame and Tumbling Frame



$$\boldsymbol{L}_{k+1}^{tp} = \boldsymbol{R}\boldsymbol{L}_{k}^{tp}$$

$$\boldsymbol{R} = \begin{bmatrix} \cos(\omega T) & 0 & -\sin(\omega T) \\ 0 & 0 & 0 \\ \sin(\omega T) & 0 & \cos(\omega T) \end{bmatrix}$$

II.B、Safety Sphere



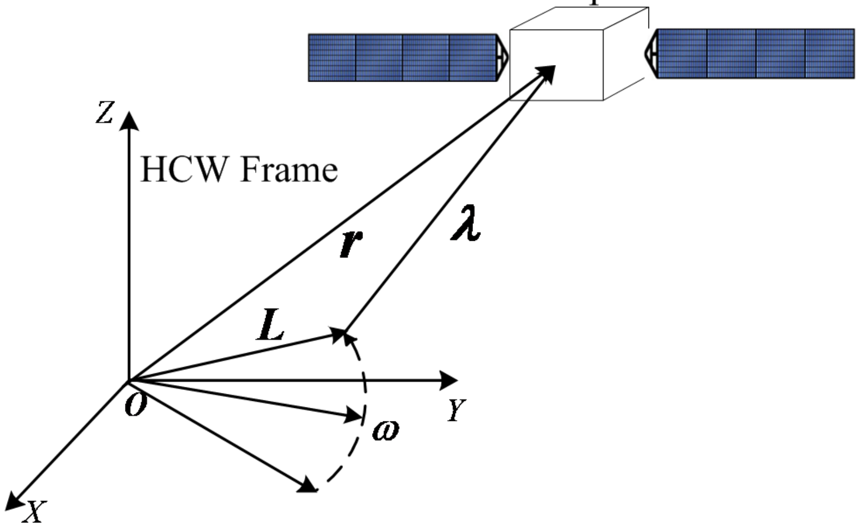
transformation from the tumbling frame to the orbital frame

Safety sphere is constructed for the chaser spacecraft to avoid collision with the flexible appendages of the target.

$$\boldsymbol{C}_{HCW}^{tp} = \boldsymbol{C}_x(\psi)\cdot\boldsymbol{C}_z(\varphi)$$

$$\boldsymbol{C}_{tp}^{I} = \boldsymbol{C}_{tp}^{HCW}\cdot\boldsymbol{C}_{HCW}^{I}$$

II.A、Relative Translation Dynamics



the relative position considering the length of the docking axis

The relative position relationships between the mass centers of the two spacecraft and the docking port can be obtained by

$$\boldsymbol{\lambda}_k = \boldsymbol{r}_k - \boldsymbol{L}_k^{HCW} = \boldsymbol{r}_k - \boldsymbol{C}_{HCW}^{tp} \boldsymbol{L}_k^{tp}$$

II.A、Relative Translation Dynamics

Define the augmented state variables in three-dimensional context as

$$\bar{\boldsymbol{X}}_k^{\alpha} = \left(\boldsymbol{r}_x, \boldsymbol{r}_y, \boldsymbol{r}_z, \dot{\boldsymbol{r}}_x, \dot{\boldsymbol{r}}_y, \dot{\boldsymbol{r}}_z, \boldsymbol{L}_x^{HCW}, \boldsymbol{L}_y^{HCW}, \boldsymbol{L}_z^{HCW}, \boldsymbol{\lambda}_x, \boldsymbol{\lambda}_y, \boldsymbol{\lambda}_z\right)^{\mathrm{T}}$$

The relative translation model can be given by

$$\bar{\boldsymbol{X}}_{k+1}^{\alpha} = \bar{\boldsymbol{A}}_d^{\alpha} \bar{\boldsymbol{X}}_k^{\alpha} + \bar{\boldsymbol{B}}_d^{\alpha} \bar{\boldsymbol{U}}_k$$

where

$$\bar{\boldsymbol{A}}_d^{\alpha} = \begin{bmatrix} \boldsymbol{A}_d^{\alpha}\big|_{6\times 6} & \boldsymbol{0}_{6\times 3} & \boldsymbol{0}_{6\times 3} \\ \boldsymbol{0}_{3\times 6} & \boldsymbol{R}_{3\times 3} & \boldsymbol{0}_{3\times 3} \\ \left[\boldsymbol{I}_{3\times 3} \quad \boldsymbol{0}_{3\times 3}\right] & -\boldsymbol{I}_{3\times 3} & \boldsymbol{0}_{3\times 3} \end{bmatrix} \qquad \bar{\boldsymbol{B}}_d^{\alpha} = \begin{bmatrix} \boldsymbol{B}_d^{\alpha}\big|_{6\times 3} \\ \boldsymbol{0}_{6\times 3} \end{bmatrix}$$

II.C、Euler Rotational Dynamics

Kinematic equations

$$\begin{bmatrix} \dot{q}_0^{\Upsilon} \\ \dot{q}_1^{\Upsilon} \\ \dot{q}_2^{\Upsilon} \\ \dot{q}_3^{\Upsilon} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -q_1^{\Upsilon}\omega_x^{\Upsilon} - q_2^{\Upsilon}\omega_y^{\Upsilon} - q_3^{\Upsilon}\omega_z^{\Upsilon} \\ q_0^{\Upsilon}\omega_x^{\Upsilon} + q_2^{\Upsilon}\omega_z^{\Upsilon} - q_3^{\Upsilon}\omega_y^{\Upsilon} \\ q_0^{\Upsilon}\omega_y^{\Upsilon} - q_1^{\Upsilon}\omega_z^{\Upsilon} + q_3^{\Upsilon}\omega_x^{\Upsilon} \\ q_0^{\Upsilon}\omega_z^{\Upsilon} + q_1^{\Upsilon}\omega_y^{\Upsilon} - q_2^{\Upsilon}\omega_x^{\Upsilon} \end{bmatrix}$$

Dynamic equations

$$\boldsymbol{J}^{\Upsilon}\dot{\boldsymbol{\omega}}^{\Upsilon} + \boldsymbol{\omega}^{\Upsilon} \times \boldsymbol{J}^{\Upsilon}\boldsymbol{\omega}^{\Upsilon} = \boldsymbol{T}^{\Upsilon}$$

The equations can be synthesized as

$$\dot{\boldsymbol{X}}^{\Upsilon} = \boldsymbol{A}^{\Upsilon}(t)\boldsymbol{X}^{\Upsilon} + \boldsymbol{B}^{\Upsilon}\boldsymbol{U}^{\Upsilon}$$

$$\begin{bmatrix} \dot{q}_0^{\Upsilon} \\ \dot{q}_1^{\Upsilon} \\ \dot{q}_2^{\Upsilon} \\ \dot{q}_3^{\Upsilon} \\ \dot{\omega}_x^{\Upsilon} \\ \dot{\omega}_y^{\Upsilon} \\ \dot{\omega}_z^{\Upsilon} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & -\omega_x^{\Upsilon} & -\omega_y^{\Upsilon} & -\omega_z^{\Upsilon} & 0 & 0 & 0 \\ \omega_x^{\Upsilon} & 0 & \omega_z^{\Upsilon} & -\omega_y^{\Upsilon} & 0 & 0 & 0 \\ \omega_y^{\Upsilon} & -\omega_z^{\Upsilon} & 0 & \omega_x^{\Upsilon} & 0 & 0 & 0 \\ \omega_z^{\Upsilon} & \omega_y^{\Upsilon} & -\omega_x^{\Upsilon} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2K_x^{\Upsilon}\omega_z^{\Upsilon} & 0 \\ 0 & 0 & 0 & 0 & -2K_y^{\Upsilon}\omega_z^{\Upsilon} & 0 & 0 \\ 0 & 0 & 0 & 0 & -2K_z^{\Upsilon}\omega_y^{\Upsilon} & 0 & 0 \end{bmatrix}\begin{bmatrix} q_0^{\Upsilon} \\ q_1^{\Upsilon} \\ q_2^{\Upsilon} \\ q_3^{\Upsilon} \\ \omega_x^{\Upsilon} \\ \omega_y^{\Upsilon} \\ \omega_z^{\Upsilon} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{J_{11}^{\Upsilon}} & 0 & 0 \\ 0 & \frac{1}{J_{22}^{\Upsilon}} & 0 \\ 0 & 0 & \frac{1}{J_{33}^{\Upsilon}} \end{bmatrix}\begin{bmatrix} T_x^{\Upsilon} \\ T_y^{\Upsilon} \\ T_z^{\Upsilon} \end{bmatrix}$$

III. SDMPC — Control Predictive Model

Key step for deriving the SPM and CPM

$$\begin{bmatrix} X_{k+1} \\ U_k \end{bmatrix} = \begin{bmatrix} A_k & B_k \\ \mathbf{0} & I \end{bmatrix} \begin{bmatrix} X_k \\ U_{k-1} \end{bmatrix} + \begin{bmatrix} B_k \\ I \end{bmatrix} \Delta U_k$$

Control Predictive Model

$$\boldsymbol{U}(k+i\,|\,k) = \sum_{i=0}^{N_c-1} \Delta \boldsymbol{U}(k+i\,|\,k) + \boldsymbol{U}(k-1)$$

Specifically

$$\begin{cases} \boldsymbol{U}(k\,|\,k) = \Delta\boldsymbol{U}(k\,|\,k) + \boldsymbol{U}(k-1) \\ \boldsymbol{U}(k+1\,|\,k) = \Delta\boldsymbol{U}(k+1\,|\,k) + \Delta\boldsymbol{U}(k\,|\,k) + \boldsymbol{U}(k-1) \\ \qquad\qquad\vdots \\ \boldsymbol{U}(k+N_c-1\,|\,k) = \Delta\boldsymbol{U}(k+N_c-1\,|\,k) + \ldots + \Delta\boldsymbol{U}(k\,|\,k) + \boldsymbol{U}(k-1) \end{cases}$$

III. SDMPC — State Predictive Model

The state prediction model (SPM) expressed by the control correction column vector $\Delta \boldsymbol{U}_c(k)$ in the prediction horizon N can be written as

$$\boldsymbol{X}_c^p(k) = \boldsymbol{\phi}\boldsymbol{X}(k) + \boldsymbol{\Gamma}\boldsymbol{U}(k-1) + \boldsymbol{G}_y \Delta \boldsymbol{U}_c(k)$$

where $\boldsymbol{X}_c^p(k)$ is a state column vector and $\boldsymbol{\phi} = \boldsymbol{A}^j, \boldsymbol{\Gamma} = \sum_{i=0}^{j-1} \boldsymbol{A}^i \boldsymbol{B}, \boldsymbol{G}_y = \left[\sum_{i=0}^{j-1} \boldsymbol{A}^i \boldsymbol{B} \quad \cdots \quad \sum_{i=0}^{j-N_c} \boldsymbol{A}^i \boldsymbol{B} \right]$.

Specifically

$$\begin{bmatrix} \boldsymbol{X}(k+1|k) \\ \vdots \\ \boldsymbol{X}(k+N_c|k) \\ \boldsymbol{X}(k+N_c+1|k) \\ \vdots \\ \boldsymbol{X}(k+N|k) \end{bmatrix} = \underbrace{\begin{bmatrix} \boldsymbol{A} \\ \vdots \\ \boldsymbol{A}^{N_c} \\ \boldsymbol{A}^{N_c+1} \\ \vdots \\ \boldsymbol{A}^{N} \end{bmatrix}}_{\boldsymbol{\phi}} \boldsymbol{X}(k) + \underbrace{\begin{bmatrix} \boldsymbol{B} \\ \vdots \\ \sum_{i=0}^{N_c-1} \boldsymbol{A}^i \boldsymbol{B} \\ \sum_{i=0}^{N_c} \boldsymbol{A}^i \boldsymbol{B} \\ \vdots \\ \sum_{i=0}^{N-1} \boldsymbol{A}^i \boldsymbol{B} \end{bmatrix}}_{\boldsymbol{\Gamma}} \boldsymbol{U}(k-1) + \underbrace{\begin{bmatrix} \boldsymbol{B} & \cdots & \boldsymbol{0} \\ \boldsymbol{AB}+\boldsymbol{B} & \cdots & \boldsymbol{0} \\ \vdots & \ddots & \vdots \\ \sum_{i=0}^{N_c-1} \boldsymbol{A}^i \boldsymbol{B} & \cdots & \boldsymbol{B} \\ \sum_{i=0}^{N_c} \boldsymbol{A}^i \boldsymbol{B} & \cdots & \boldsymbol{AB}+\boldsymbol{B} \\ \vdots & \vdots & \vdots \\ \sum_{i=0}^{N-1} \boldsymbol{A}^i \boldsymbol{B} & \cdots & \sum_{i=0}^{N-N_c} \boldsymbol{A}^i \boldsymbol{B} \end{bmatrix}}_{\boldsymbol{G}_y} \begin{bmatrix} \Delta \boldsymbol{U}(k|k) \\ \vdots \\ \Delta \boldsymbol{U}(k+N_c-1|k) \end{bmatrix}$$

III. SDMPC — Cost Function

The cost function for the SDMPC is designed to penalize the error between the predictive states and the reference states in the predictive horizon, while to minimize the fuel consumption in the control horizon. The cost function at the time instant k is given by

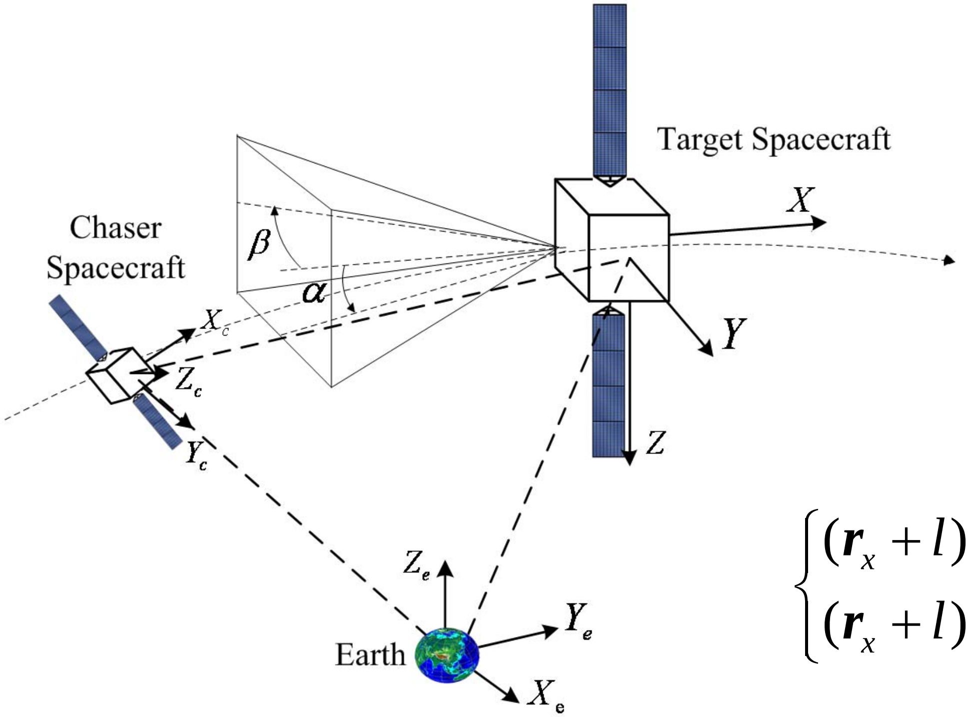
$$J_k = \sum_{j=1}^{N} \left\| \boldsymbol{X}(k+j\,|\,k) - \boldsymbol{X}_{ref}(k+j\,|\,k) \right\|_{\boldsymbol{Q}}^2 + \sum_{i=0}^{N_c-1} \left\| \Delta \boldsymbol{U}(k+i\,|\,k) \right\|_{\boldsymbol{R}}^2$$

Transformation

$$J_k = \frac{1}{2} \Delta \boldsymbol{U}_c^{\mathrm{T}}(k) \boldsymbol{H} \Delta \boldsymbol{U}_c(k) + \boldsymbol{f}^{\mathrm{T}} \Delta \boldsymbol{U}_c(k) + const$$

Now, the problem has been transformed into a standard QPP which can be solved by using Matlab

III. SDMPC — LOS Constraint



$$
\begin{cases}
-\tan\alpha \le \dfrac{\boldsymbol{r}_z}{\boldsymbol{r}_x + l} \le \tan\alpha \\
-\tan\beta \le \dfrac{\boldsymbol{r}_y}{\boldsymbol{r}_x + l} \le \tan\beta
\end{cases}
$$

$$
\begin{cases}
(\boldsymbol{r}_x + l)\tan\alpha + \boldsymbol{r}_z \le 0, (\boldsymbol{r}_x + l)\tan\alpha - \boldsymbol{r}_z \le 0 \\
(\boldsymbol{r}_x + l)\tan\beta + \boldsymbol{r}_y \le 0, (\boldsymbol{r}_x + l)\tan\beta - \boldsymbol{r}_y \le 0
\end{cases}
$$

III. SDMPC — Control Actuators Magnitude Constraint

Whatever for the powered thrusters equipped in the orbit control system (OCS) or small thrusters and reaction wheels equipped in the attitude control system (ACS), the output capacity of a real spacecraft should be restricted to a reasonable level. The control outputs are bounded by

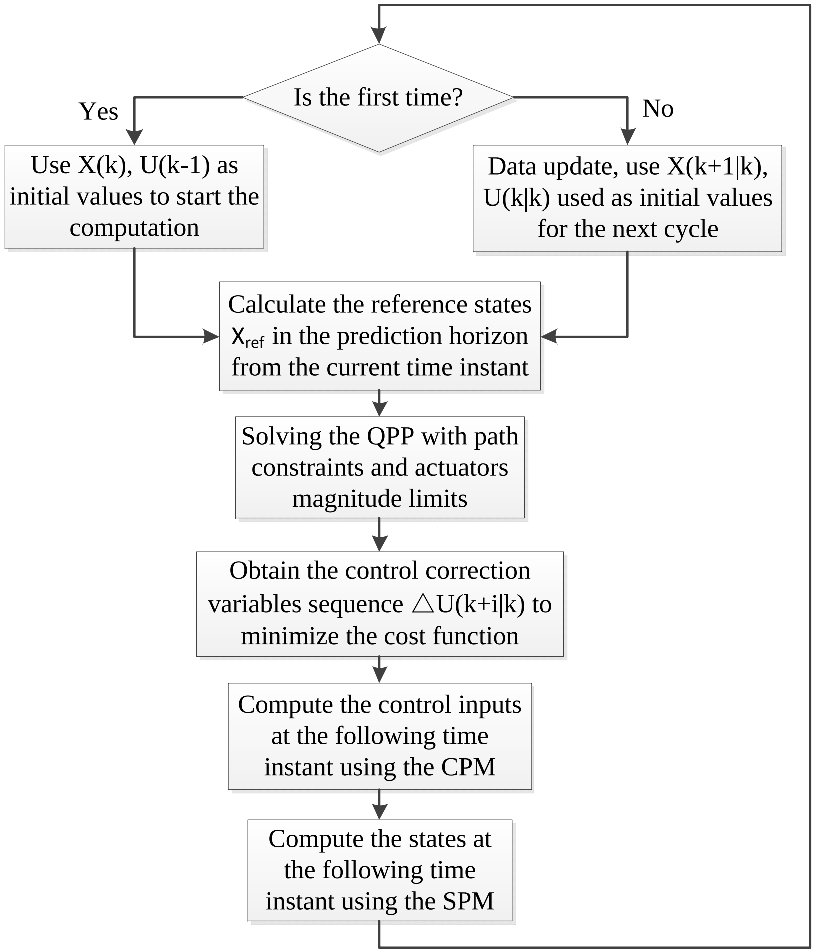
$$\boldsymbol{U}^{min} \le \boldsymbol{U}(k+i \mid k) \le \boldsymbol{U}^{max}, (i = 0, \ldots, N_c - 1)$$

$$\begin{bmatrix} \boldsymbol{M} \\ -\boldsymbol{M} \end{bmatrix} \Delta \boldsymbol{U}_c(k) \le \begin{bmatrix} \boldsymbol{U}_c^{max} - \boldsymbol{F}\boldsymbol{U}(k-1) \\ -\boldsymbol{U}_c^{min} + \boldsymbol{F}\boldsymbol{U}(k-1) \end{bmatrix}$$

where

$$\boldsymbol{M} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{I} & \boldsymbol{I} & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{I} & \boldsymbol{I} & \cdots & \boldsymbol{I} \end{bmatrix} \quad \boldsymbol{F} = \begin{bmatrix} \boldsymbol{I} \\ \boldsymbol{I} \\ \vdots \\ \boldsymbol{I} \end{bmatrix} \quad \boldsymbol{U}_c^{max} = \begin{bmatrix} \boldsymbol{U}^{max} \\ \boldsymbol{U}^{max} \\ \vdots \\ \boldsymbol{U}^{max} \end{bmatrix} \quad \boldsymbol{U}_c^{min} = \begin{bmatrix} \boldsymbol{U}^{min} \\ \boldsymbol{U}^{min} \\ \vdots \\ \boldsymbol{U}^{min} \end{bmatrix}$$

III. SDMPC — Receding Horizon Optimization



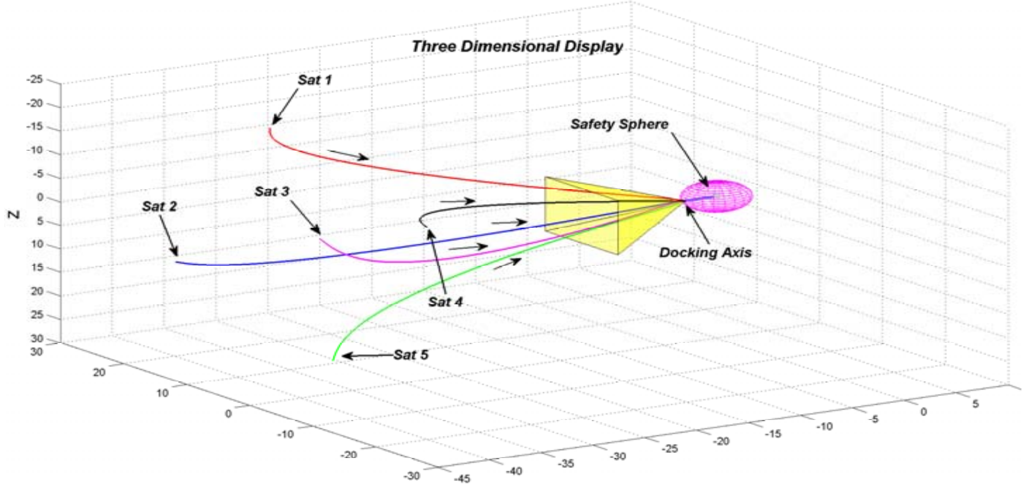
① The SPM is used to predict the future the outputs, based on past and current values.

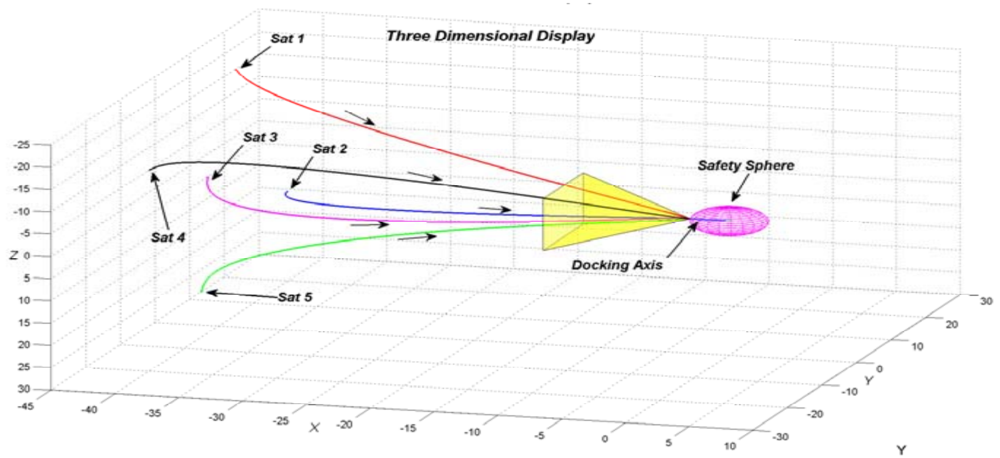
② These actions are calculated by the optimizer taking into account the cost function (where the fuel cost and the future tracking error are considered) as well as the constraints.

③ The solution is obtained by computing a QPP on the control prediction horizon.

④ The first control input in the sequence is then applied to the plant, and the optimization is repeated with the new initial conditions.

IV. Numerical Results —Approaching a Stationary Target



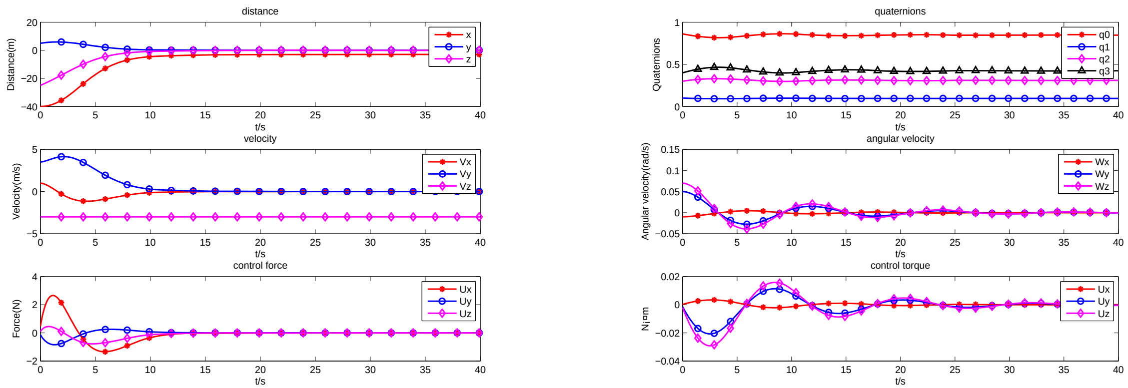


The direction of the docking axis vector is from the mass center of the target to the docking port, which is assumed to point to the opposite direction of the velocity of the target. Hence, the chaser needs to rendezvous with the target along the V-bar direction.

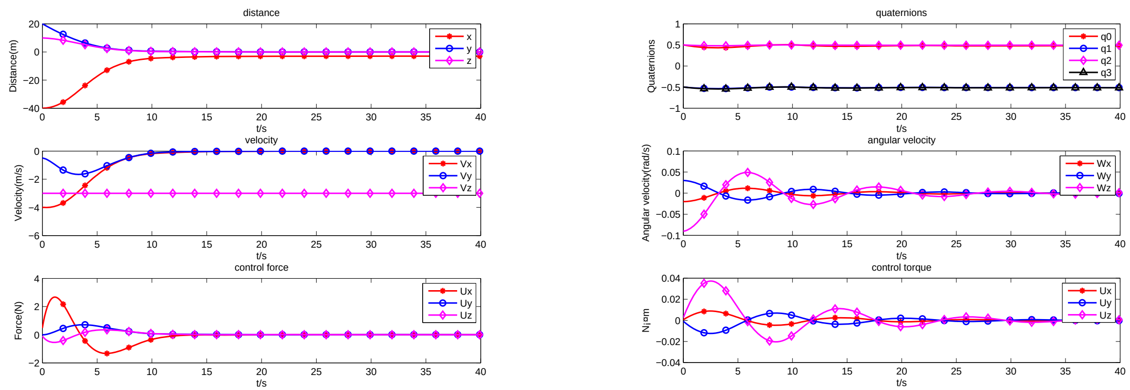
However, the LOS tetrahedral path constraint applied here is also suitable for approaching from the R-Bar and H-Bar direction.

In order to verify the effectiveness of the SDMPC algorithm, a comparison is made by five chaser vehicles which start the autonomous rendezvous from different positions with different initial velocities.

IV. Numerical Results ——Approaching Stationary Target

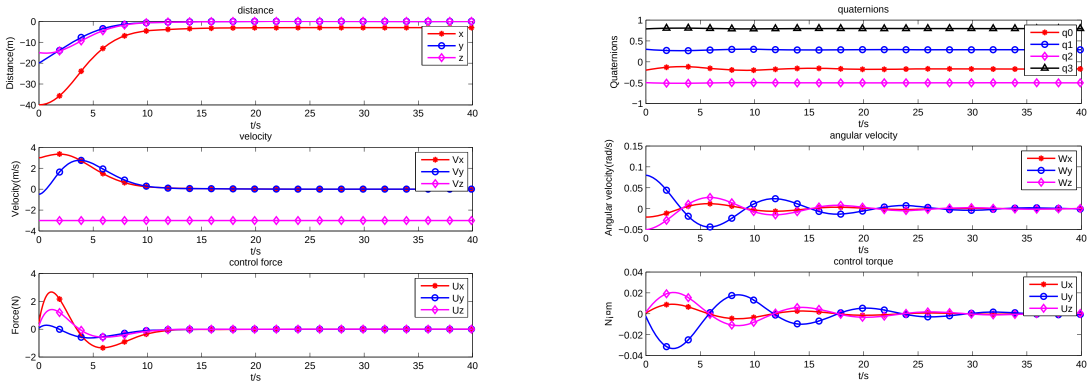


Relative motion states of Sat 1

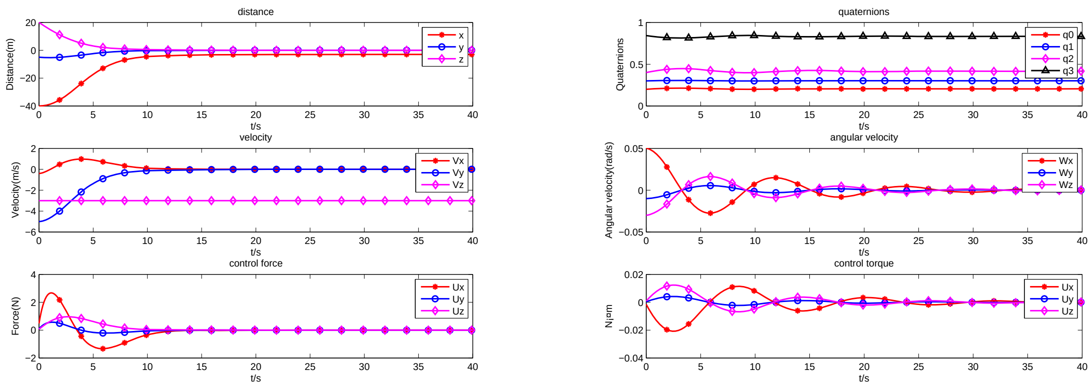


Relative motion states of Sat 2

IV. Numerical Results ——Approaching Stationary Target

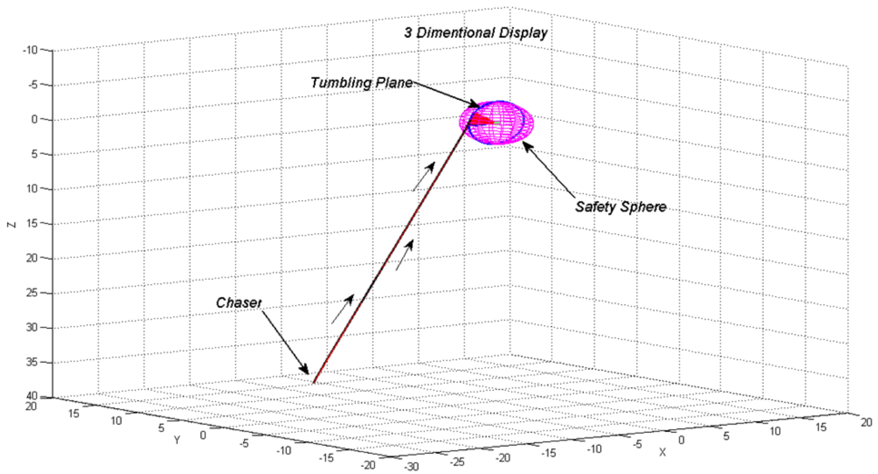


Relative motion states of Sat 4

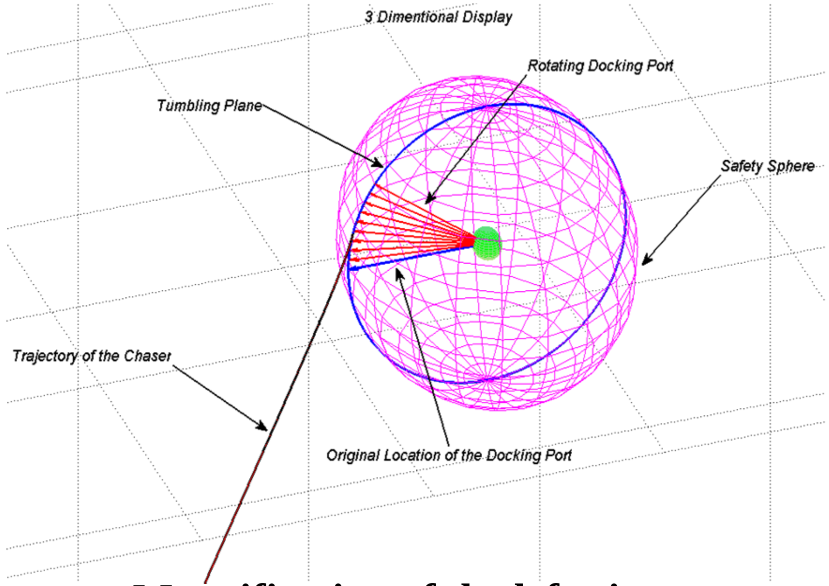


Relative motion states of Sat 5

IV. Numerical Results —Approaching Tumbling Target

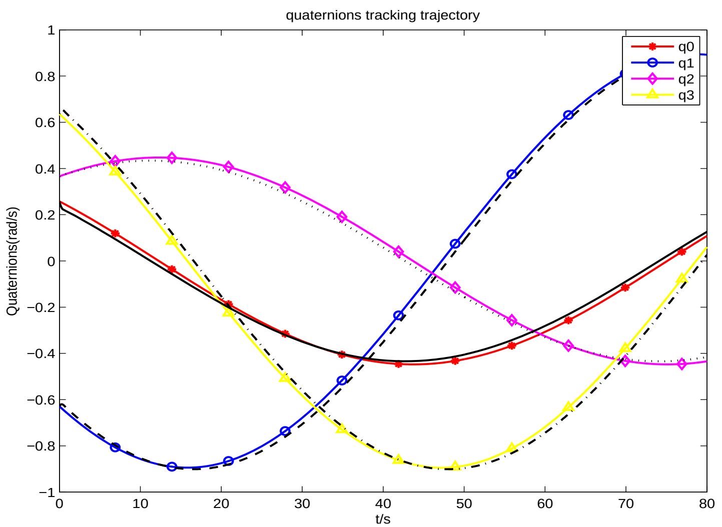


3 dimensional display of approaching a tumbling target

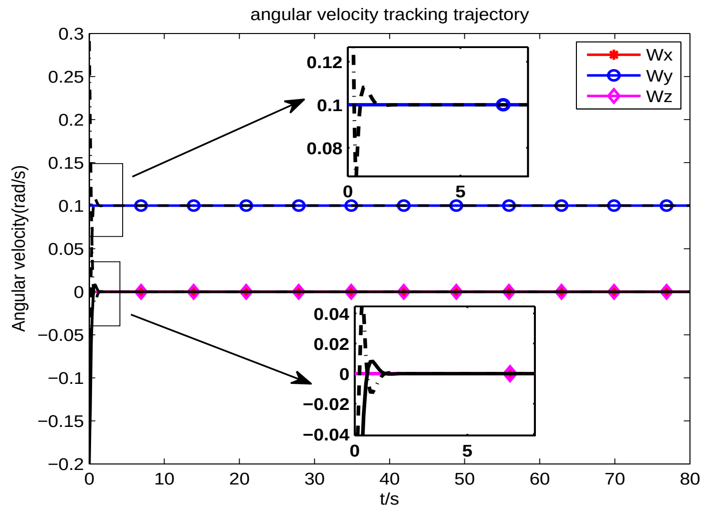


Magnification of the left picture

IV. Numerical Results ——Approaching Tumbling Target



Quaternions



Angular velocity

As can be seen from the pictures, the quaternions track the time-varying values closely and the angular velocity converges to the ideal angular velocity within less than 10 seconds. Meanwhile, the control torque is moderate and can be satisfied by the current reaction wheels or small-output thrusters.

V. Conclusions

- **Conclusions**
  - A 26-state model of a two-spacecraft rendezvous is presented
  - The optimal control problems are formulated and addressed using the SDMPC control methods
  - Both scenarios of approaching a three-axis attitude stabilized target and a tumbling target are considered and the simulations are implemented. The results obtained show the good performance of the SDMPC.
- **Future work**
  - TH equations should be used to describe the relative motion in a highly elliptical orbit, and the influence of the eccentricity on the performance of this algorithm needs to be studied.
  - As essential parameters in the SDMPC, the effects of prediction horizon and control horizon on the fuel consumption should be investigated.

Thanks for your attention !