OPTIMAL COORDINATED ATTITUDE CONTROL OF MULTIPLE SATELLITES IN A FORMATION

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Abstract: *The focus of the research on satellite formation flying has so far been on the coordinated satellite orbit control. A number of future satellite formations will however require the ability to achieve coordinated attitude control between multiple satellites. Such missions will mainly require two relative attitude control modes: attitude synchronization and target tracking. Examples of such satellite formations are those that emulate the functions of large space telescopes or space interferometers. In this paper, an optimal SDRE (state Dependent Riccati Equation) controller is proposed to coordinate the attitude synchronization ensuring that the relative orientations between the satellites are controlled whilst tracking a ground target.*

Keywords: *Coordinated attitude control, satellite formation, optimal, SDRE.*

1. Introduction

Satellite formations are being considered for an increasingly wide range of applications from the observation of Earth's magnetosphere, gravimetry missions to astronomy and deep space interferometry. Certain space missions that would have in theory required very large monolithic spacecraft have now been made possible at a much lower cost using the formation flying concept. To date, the research focus in formation flying has been on relative navigation and relative orbit control, by considering orbits enabling interferometry or other formation flying applications with conventional, generally nadir pointing, attitude control modes. This has been the case with the technology demonstration type of missions, often consisting of two satellites, such as the Prisma mission and the planned Proba-3 mission, where relative navigation instruments and new propulsion technologies are being validated. An exception was the Topsat SAR mission design which considered relative yaw attitude synchronisation.

The increasing requirements on future space missions with two or more satellites are making relative attitude determination and control a research problem to be addressed. Two relative attitude control modes are of particular relevance to future space missions: Target tracking to a master spacecraft from multiple satellites and attitude synchronisation of multiple satellites. The first mode is useful to download data to the master satellite that would communicate with the ground, while the second mode would typically be required during science observations, where relative attitude control is required to synchronise the satellites attitudes. For the formation to operate as a unit, such as a telescope or a SAR instrument, attitude synchronisation must be accurately maintained during inertial pointing or target tracking of an orbiting satellite formation (see figure 1). In theory, each satellite could independently control its absolute attitude, but feedback control of the relative attitude is required in practice because the satellites experience different disturbances, uncertainties, making accurate attitude synchronisation impossible without relative attitude feedback.

Attitude synchronisation requires specific hardware and software for the estimation of the relative attitude. Estimated attitudes can theoretically be communicated between the satellites during relative attitude control. [1], [2], [3] This however imposes the maintenance of intersatellite links throughout the manoeuvres and causes robustness issues due to communication delays.

Two conventional relative navigation techniques can be applied to relative attitude determination: Visual navigation, and DGPS (Differential Global Positioning System). Visual navigation can indeed be considered for the attitude synchronisation mode under specific formation configurations. A target tracking scenario is the best suited for relative attitude determination by visual navigation sensors. [8],[9],[10]. DGPS is another solution for attitude determination from arbitrary orbits to perform attitude synchronisation is the use of the carrier phase DGPS technique. While DGPS is better known for providing relative positions, the use of a pair or more of GPS receivers on the same can provide the attitude angle, by matching the SNR pattern with antennae gain patterns. [11], [12] Recently, Lightsey et al. was the first to propose the use of DGPS for relative attitude determination in a satellite formation, shortly followed by Buist et al. [13], [14] It is in fact for satellite formations that such new attitude determination techniques are needed. Relative attitude determination is beyond the scope of this paper, where it is assumed that relative attitudes are known, whether from intersatellite communication, the use of multiple DGPS receivers or optical navigation. The focus of the paper is on the relative attitude control problem with multiple satellites.

Two competing control objectives have to be considered when formulating a relative attitude control problem for formation flying: Station keeping and formation keeping. Satellites would generally have to maintain their relative attitudes with respect to each other, while the formation manoeuvres towards a direction of interest for scientific observations or also for communication with a ground station. Both absolute and relative attitudes are therefore fed back to each satellite. In a fully centralised approach, each satellite requires all relative attitudes of other satellites. In the decentralised approach under consideration here, each satellite only requires relative attitudes of other satellites with itself, as well as the absolute attitude. Absolute attitude information alone would not be sufficient to maintain relative attitude control performance during science observation of formation manoeuvres.

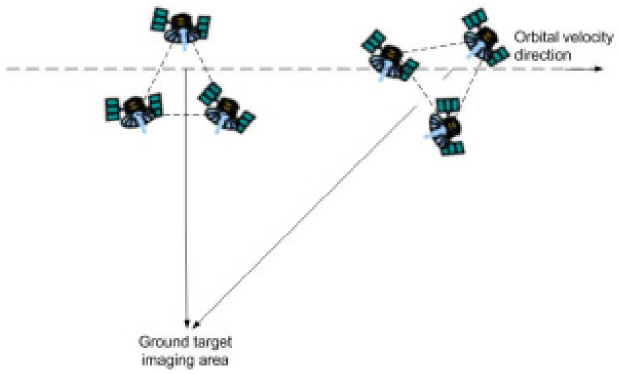

Figure 1: Example of a synchronised relative attitude scenario

The traditional approach to the attitude synchronisation problem was the centralised nearest neighbour tracking approach, where a leader controls its absolute attitude and all satellites in the formation correct their relative attitudes with respect to it. The issue with that approach is a major dependency on the success of the leader's attitude tracking in this centralised approach. A decentralised approach presents the advantage of having no leader and that one satellite failure can easily be resolved by reducing the order of the formation and adopting the exact same control approach, with no need to reassign a new leader.

Vandyke and Hall were the first authors to design a decentralised stabilising controller that simultaneously drives the absolute and relative attitude errors to zero. [1] The controller was a quaternion feedback law, which is practically useful because quaternions are readily available in the onboard software of most satellites. A robust stabilising coordinated attitude controller was then proposed for the same formation flying problem by Erdong, Xiaoleib and Zhaoweia. [2] In that reference, a sliding mode controller was designed and shown to achieve robust stability in the presence of intersatellite communication delays and severe external disturbances. The first decentralised optimal controller was then proposed by Chang, Park and Choi based on an SDRE (State Deopendent Riccati Equation) approach, which involves solving an LQR (Linear Quadratic Regulation) optimal control problem at each new state. The SDRE approach presents the advantage of applying to the nonlinear model of attitude dynamics. [3] In reference [3], SDRE controllers were applied separately to the absolute and relative attitude control problems, which made proving stability a simple task, but the issue is that the optimal solution to achieving a trade-off between station keeping and formation keeping is not as trivial as adding the optimal solutions of both. There is indeed a coupling between the two and the sum of two optimal laws is not the optimal solution to the sum of the cost functions, when there is a coupling in the problem.

In this paper, an SDRE optimal control approach is proposed to achieve and optimal tradeoff between formation keeping and stationkeeping by constructing a single cost functional with a penalty on both station keeping and formation keeping. Section 2 describes the mathematical models of station keeping and formation keeping for the coordinated attitude control in a satellite formation. Sections 3 and 4 respectively describe the controllers of reference [1] and [3]. The new SDRE optimal coordinated attitude control approach is presented in section 5 and numerical simulations demonstrate the performance enhancement by this approach in section 6.

2. Dynamic and kinematic models:

The dynamic model for each satellite is given by Euler's dynamic equation. The kinematic model is expressed in terms of quaternion parameters. The equations for the absolute and relative attitude dynamics and kinematics are given by:

$$\begin{cases} \mathbf{I}_i\dot{\boldsymbol{\omega}}_\mathrm{i} = \mathbf{u}_i - \boldsymbol{\omega}_\mathrm{i} \times (\mathbf{I}_i\boldsymbol{\omega}_\mathrm{i} + \mathbf{h}_i), \mathbf{u}_i = -\dot{\mathbf{h}}_i \\ \dot{\mathbf{q}}_i = -\frac{1}{2}\boldsymbol{\omega}_i^{\times}\mathbf{q}_i + \frac{1}{2}q_{4i}\boldsymbol{\omega}_i \\ \dot{q}_{4i} = -\frac{1}{2}\boldsymbol{\omega}_i^{\mathrm{T}}\mathbf{q}_i \end{cases} \quad (1)$$

$$\begin{cases} \boldsymbol{\omega}^i_j = \boldsymbol{\omega}^i_I - \mathbf{R}^i_j\boldsymbol{\omega}^j_I \\ \dot{\mathbf{q}}_{ij} = -\frac{1}{2}\boldsymbol{\omega}_{ij}^{\times}\mathbf{q}_{ij} + \frac{1}{2}q_{4ij}\boldsymbol{\omega}_{ij} \\ \dot{q}_{4ij} = -\frac{1}{2}\boldsymbol{\omega}_{ij}^{\mathrm{T}}\mathbf{q}_{ij} \end{cases} \quad (2)$$

where $\boldsymbol{\omega}_i$, $\boldsymbol{\omega}_{ij}$ denote the absolute and relative body angular velocities, $\mathbf{q}_i$, $\mathbf{q}_{ij}$ denote the absolute and relative attitude quaternions, $\dot{\mathbf{h}}_i$ is the control torque vector of satellite i. The system of equations can also be written as a state space model where the state vector is $\mathbf{x}_1 = [\mathbf{q}_1,\ldots\mathbf{q}_{ij}\ldots,\dot{\mathbf{q}}_1,\ldots\dot{\mathbf{q}}_{ij}\ldots]^T, j \neq i$, i=1, N includes the absolute and relative attitudes of each satellite i, the control torque vector and $\mathbf{u}_i$ is the control vector of the wheels on satellite i.

3. PD type State feedback control approach

The first controller proposed by Vandyke and Hall to achieve the competing objectives of formation keeping and station keeping was a PD type quaternion feedback controller and the control law was given by:

$$\mathbf{u}_i = -k_{pi}\mathbf{q}_i - k_{di}\boldsymbol{\omega}_i - \sum_{j=1}^{N} c_p^{ij}\mathbf{q}^i_j - \sum_{j=1}^{N} c_d^{ij}\boldsymbol{\omega}^i_j \quad (3)$$

The paper provided a stability proof together with an analysis of the dependency of the overall control torque consumption and of steady state tracking error metrics on the number of interconnections per satellite (or the number of relative attitudes fed back to each satellite.

4. SDRE control approach proposed in the literature

In reference [2], continuous SDRE feedback was proposed by separately solving the optimal station keeping and formation keeping problems:

$$J_i = \int_0^\infty \left(\mathbf{x}_i^{\mathrm{T}} \mathbf{Q}_i \mathbf{x}_i + \mathbf{u}_i^{\mathrm{T}} \mathbf{R}_i \mathbf{u}_i \right) dt$$

$$J_{ij} = \int_0^\infty \left(\mathbf{x}_{ij}^{\mathrm{T}} \mathbf{Q}_{ij} \mathbf{x}_{ij} + \mathbf{u}_{ij}^{\mathrm{T}} \mathbf{R}_{ij} \mathbf{u}_{ij} \right) dt \tag{3}$$

The final control input was then obtained by simply superposing (adding) the optimal solution of absolute attitude control $\mathbf{u}_i$ to the optimal solution of relative attitude control $\mathbf{u}_{ij.}$ The sum of the two did guarantee stability of the overall system, but the issue was that adding two optimal contributions does not amount to an optimal solution to the overall cost with a penalty on both station keeping and formation keeping, because of the coupling between the two.

5. The new SDRE optimal control approach

The limitation of this approach is that adding the optimal station keeping solution to the optimal formation keeping solution does not guarantee an overall optimal tradeoff. While this solution achieves stability and achieves reasonable performance, it is more convenient to directly formulate the optimization problem as a weighted tradeoff between station keeping and formation keeping performance, as proposed here:

With:

$$\begin{aligned} J_i &= \int_0^\infty \left(\mathbf{y}_i^{T} \mathbf{W}_{yi} \mathbf{y}_i + \mathbf{u}_i^{T} \mathbf{R}_i \mathbf{u}_i \right) dt \\ &= \int_0^\infty \left(\mathbf{x}_i^{T} \mathbf{C}_i^{T} \mathbf{W}_{yi} \mathbf{C}_i \mathbf{x}_i + \mathbf{u}_i^{T} \mathbf{R}_i \mathbf{u}_i \right) dt \end{aligned} \tag{4}$$

which is a quadratic cost function where $\mathbf{u}_i$ is the control input vector of satellite i (i=1,N), $\mathbf{x}_i = [q_i,...q_{ij,....}, \dot{q}_i,...\dot{q}_{ij}....]^T$ is the state vector of satellite i and the state weighting matrix is given by:

$$\mathbf{Q}_i = \mathbf{C}_i^T \mathbf{W}_{yi} \mathbf{C}_i \tag{5}$$

and the output $\mathbf{y}_i$ is given by:

$$\mathbf{y}_i = \mathbf{C}_i \mathbf{x}_i = \mathbf{C}_i [q_i,...q_{ij,....}, \dot{q}_i,...\dot{q}_{ij}....]^T \cdot \quad j \neq i, i = 1, N$$

For example, with $i = 1 : \mathbf{y}_1 = \mathbf{C}_1 \mathbf{x}_1 = [q_1, q_{12}, ..., q_{1n}, \dot{q}_1, \dot{q}_{12}, ..., \dot{q}_{1n}]^T$ and the first output being minimised incorporates the absolute attitude and attitude rate of satellite 1 as well as the relative attitudes and attitude rates between satellite 1 and the other satellites in the formation.

The observation matrix is given by:

$$\mathbf{C}_1 = \begin{pmatrix} I_{3\times3} & Q_2 & .. & Q_n & 0 & \dot{Q}_2 & .. & \dot{Q}_n \\ 0 & 0 & .. & 0 & I_{3\times3} & Q_2 & .. & Q_n \end{pmatrix}^T_{6\times 6N}$$

The matrices $\mathbf{C}_2$, …, $\mathbf{C}_N$ are defined similarly by shifting the identity matrix to columns 2,.., N , and N+2,…,2N, where N is the number of satellites.

The SDRE controller is obtained by solving an LQR problem (Riccatti equation) at each new state $\mathbf{x}_i$:

$$\mathbf{A}_i^T(\mathbf{x}_i)\mathbf{P}_i(\mathbf{x}_i)+\mathbf{P}_i(\mathbf{x}_i)\mathbf{A}_i(\mathbf{x}_i)-\mathbf{P}_i(\mathbf{x}_i)\mathbf{B}_i(\mathbf{x}_i)\mathbf{R}_i^{-1}(\mathbf{x}_i)\mathbf{B}_i^T(\mathbf{x}_i)\mathbf{P}(\mathbf{x}_i)+\mathbf{Q}_i=0 \tag{6}$$

The controller gain matrix is then given by:

$$\mathbf{K}_i(\mathbf{x}_i)=\mathbf{R}_i^{-1}(\mathbf{x}_i)\mathbf{B}_i^T(\mathbf{x}_i)\mathbf{P}_i(\mathbf{x}_i) \tag{7}$$

and the control torque of satellite i is given by:

$$\mathbf{u}_i=-\mathbf{K}_i(\mathbf{x}_i)\mathbf{x}_i \tag{8}$$

The approach has low computational demand (matrix sizes are reduced using a decentralized control approach, the number of SDRE blocks is equal to N (number of satellites). It is independent from the number of interconnections that would have had to be taken into consideration in a centralized control approach.

Each satellite achieves a trade-off between station keeping and formation keeping with other satellites, which is weighted by the matrix $\mathbf{W}_i$.

6. Numerical simulations:

Initial simulations of the absolute and relative attitude control with the proposed SDRE approach were performed for a microsatellite with moments of inertia I = diag [10,14, 12], corresponding to the parameters of a 100 kg class microsatellite. A formation of 4 satellites is considered assuming that relative attitudes are exchanged as shown in figure 2.

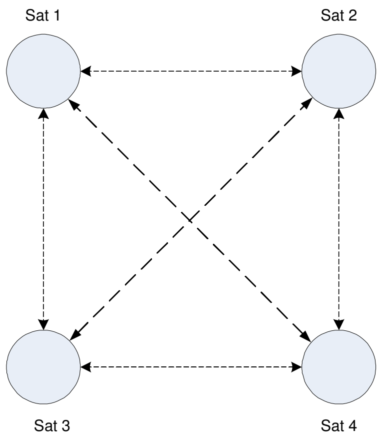

Figure 2. Topology of the relative attitudes in the assumed formation of 4 satellites

The attitude control performance was compared to the approach used in reference [1] with similar overall torque expenditure. The scalar part of the quaternion was simulated for both absolute and relative attitudes, because it gives a measure of the 3-axis attitude error with respect to the desired (zero inertial pointing) orientation. Inertial pointing was considered here for simplicity but nadir pointing is also possible by defining attitude quaternion errors with respect to nadir. The attitude error is the difference between that scalar quaternion and 1. Figure (3) shows that the attitude reference is tracked significantly faster using the proposed decentralized SDRE approach, compared to the approach used in reference [1], the difference in settling time within +/-2% of the manoeuvre's amplitude ranged between 5.43 seconds and 32.52 seconds depending on the satellite. The controllers were tuned to consume equal amounts of integrated torque, within a tolerance. The control torque of the SDRE controller is shown in figure (4) to follow a smooth profile. The maximum instantaneous torque is shown in figure (4) to be admissible, not exceeding 60 mNm, which is within the capabilities of a 100 kg class microsatellite. Four (04) SDRE blocks, implying four parallel Riccati solvers, were sufficient for this scenario involving 4 satellites.

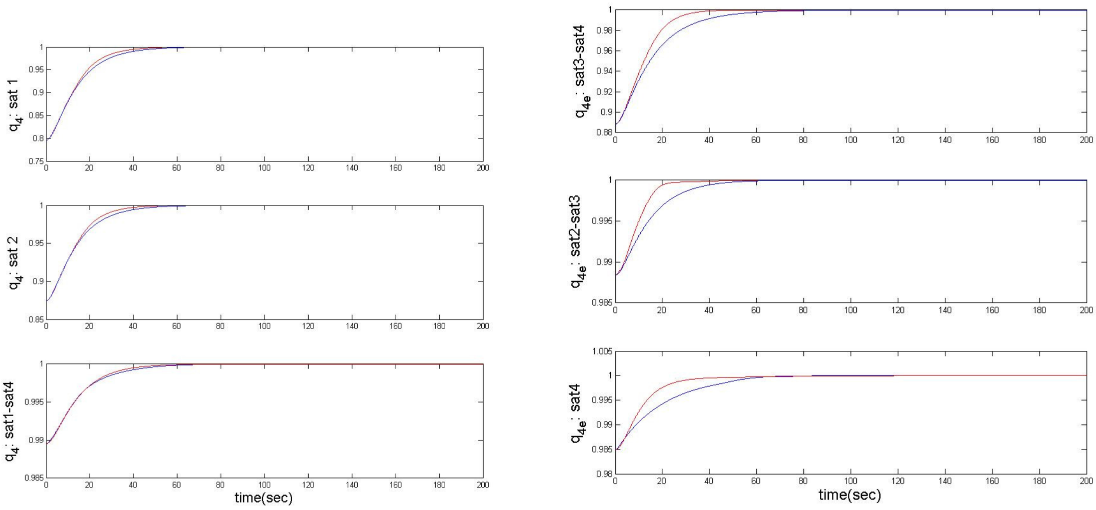

Figure 3. Absolute and relative attitude histories (formation of 4 satellies) using the proposed SDRE approach (red) and the PD type feedback (blue) of reference [1]

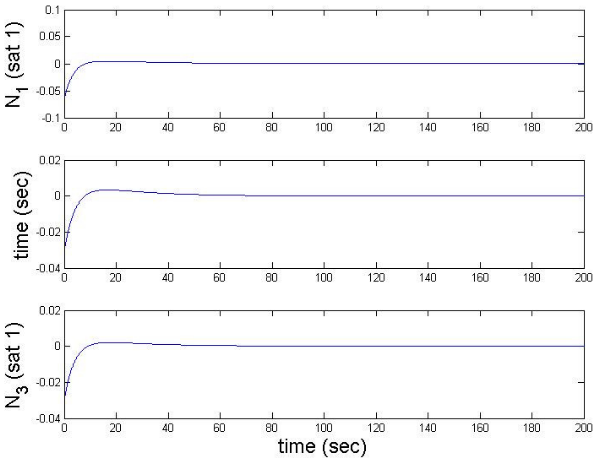

Figure 4. Time history of the control torque using the proposed SDRE controller

7. Conclusion:

A new SDRE optimal controller was designed to achieve an optimal trade-off between station keeping and formation keeping for coordinated attitude control in a formation of multiple satellites. Early numerical simulations show a significant performance trade-off enhancement compared to a conventional PD type state feedback controller. The approach is simple to implement as it simply requires the parallel implementation of standard LQR solvers, the number of which is equal to the number of satellites in the formation. This attitude control approach requires each satellite to be fed back its absolute attitude as well as its relative attitudes in real time with respect to other satellites in the formation. The control approach is decentralised by making the attitude control of a satellite independent from the relative attitudes between other satellites.

8. References

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