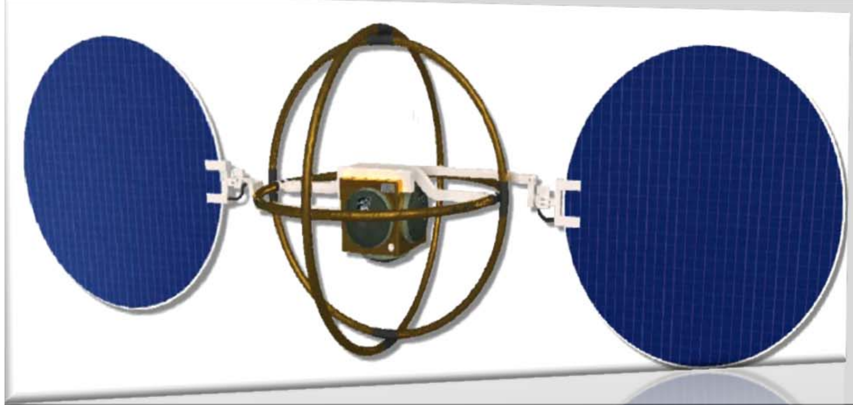

OPTIMAL CONTROL OF 6-DOF ELECTROMAGNETIC FORMATION USING THE LEGENDRE PSEUDOSPECTRAL METHOD

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- Formation flying is an enabling technology. Electromagnetic Formation Flying (EMFF) uses HTS coils to provide forces and torque, in exchange of a highly **coupled** and **nonlinear** dynamics model.

- The **coupling** effects:
  - Both magnitude and orientation of the Electromagnetic (EM) force is determined by magnetic dipole strength and relative DOF of the array.
  - When a shear EM force acts, a shear torque is introduced.

➢ Far-Field Model

- EM force

$$\mathbf{F}_{ij} = \frac{3\mu_0}{4\pi}[-\frac{5(\boldsymbol{\mu}_i \cdot \mathbf{r}_{ij})(\boldsymbol{\mu}_j \cdot \mathbf{r}_{ij})}{r_{ij}^7}\mathbf{r}_{ij} + \frac{(\boldsymbol{\mu}_i \cdot \mathbf{r}_{ij})}{r_{ij}^5}\boldsymbol{\mu}_j + \frac{(\boldsymbol{\mu}_j \cdot \mathbf{r}_{ij})}{r_{ij}^5}\boldsymbol{\mu}_i + \frac{(\boldsymbol{\mu}_i \cdot \boldsymbol{\mu}_j)}{r_{ij}^5}\mathbf{r}_{ij}]$$

$$\mathbf{F}_{ji} = -\mathbf{F}_{ij}$$

- EM torque

$$\mathbf{T}_{eij} = \frac{\mu_0 \boldsymbol{\mu}_i}{4\pi} \times [\frac{3\mathbf{r}_{ij}(\boldsymbol{\mu}_j \cdot \mathbf{r}_{ij})}{r_{ij}^5} - \frac{\boldsymbol{\mu}_j}{r_{ij}^3}]$$

$$\mathbf{T}_{eji} = \frac{\mu_0 \boldsymbol{\mu}_j}{4\pi} \times [\frac{3\mathbf{r}_{ij}(\boldsymbol{\mu}_i \cdot \mathbf{r}_{ij})}{r_{ij}^5} - \frac{\boldsymbol{\mu}_i}{r_{ij}^3}]$$

➢ Four-Satellite Planar Formation

- Assuming that the distribution of the four-satellite planar formation on the General Circular Orbit (GCO) is **symmetrical** with respect to the origin of the LVLH coordinate.

- Assuming that the **equivalent magnet moment** and maneuver trajectories are **rotational symmetrical**, satellites under similar dynamical circumstance are accordant to each other and can be handled in chorus.

➢ Four-Satellite Planar Formation

$$\mathbf{F}_A = \mathbf{F}_A(\mu_A, r_A, \alpha_A, \theta_A)$$

μ_A : the dipole strength;

α_A: the deflection angle of dipole moment in the local frame;

r_A : the radius of transition orbit; θ_A : the phase angle in GCO.

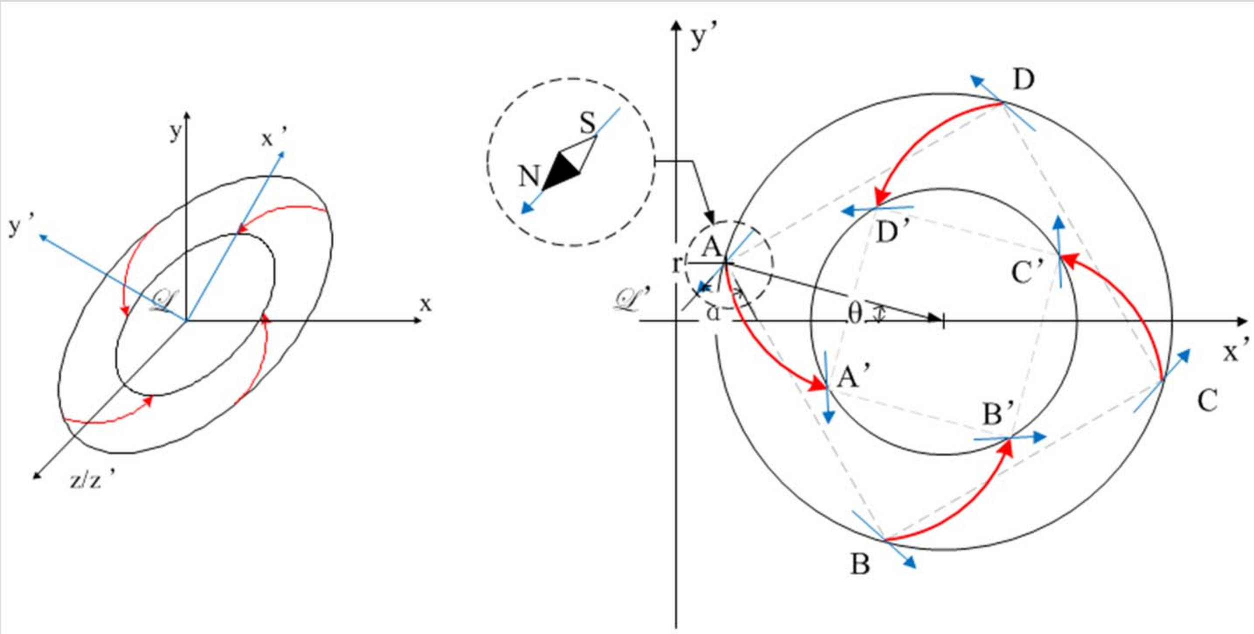


Fig 3. Ideal scheme for relative orbit transfer

➢ Relative Translational Dynamics

$$\ddot{\mathbf{r}} = \mathbf{A}_1\mathbf{r} + \mathbf{A}_2\dot{\mathbf{r}} + \mathbf{a}$$

where

$$\mathbf{A}_1 = \begin{bmatrix} \dot{\theta}^2 + 2\dfrac{\mu}{r_1^3} & \ddot{\theta}^2 & 0 \\ \ddot{\theta} & \dot{\theta}^2 - \dfrac{\mu}{r_1^3} & 0 \\ 0 & 0 & \dfrac{\mu}{r_1^3} \end{bmatrix} \qquad \mathbf{A}_2 = \begin{bmatrix} 0 & 2\dot{\theta} & 0 \\ -2\dot{\theta} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- Both state and control vector need to be **normalized** to guarantee same magnitude.

➢ Relative Rotational Dynamics

- Describing it in the i[th] deputy satellite's body-fixed frame.

$$\begin{bmatrix} \ddot{q}_0 \\ \ddot{\mathbf{q}}_v \end{bmatrix} = \begin{bmatrix} -\frac{1}{4}(\varpi^{\mathrm{T}}\varpi)q_0 - \frac{1}{2}\mathbf{q}_v^{\mathrm{T}}\mathbf{f} - \frac{1}{2}\mathbf{q}_v^{\mathrm{T}}\mathbf{I}^{-1}\mathbf{T}_\Delta \\ -\frac{1}{4}(\varpi^{\mathrm{T}}\varpi)\mathbf{q}_v + \frac{1}{2}\mathbf{Q}_v\mathbf{f} + \frac{1}{2}\mathbf{Q}_v\mathbf{I}^{-1}\mathbf{T}_\Delta \end{bmatrix}$$

where $\mathbf{T}_\Delta = (\mathbf{T}_{ei} + \mathbf{T}_{ci}) - \mathbf{I}\mathbf{A}_{ic}\mathbf{I}^{-1}(\mathbf{T}_{ec} + \mathbf{T}_{cc})$, defined as **the equivalent combined control moment**. It contains EM torque and RWs moment acted on both satellites and can be optimized to allocate the angular momentum.

- The coupling among EMFF is reflected implicitly in the connection between EM force and torque.

➢ 6-DOF Relative Dynamics Model

- Selecting $\mathbf{x}=[\mathbf{r}^{\mathrm{T}},\mathbf{q}_v^{\mathrm{T}}]^{\mathrm{T}}$ and $\mathbf{u}=[\mathbf{a}^{\mathrm{T}},\mathbf{T}_\Delta{}^{\mathrm{T}}]^{\mathrm{T}}$ as state and control variables.

$$\ddot{\mathbf{x}} = g(\mathbf{x},\dot{\mathbf{x}}) + \mathbf{C}\mathbf{u}$$

$$\mathbf{B}_1 = -\frac{1}{4}(\varpi^{\mathrm{T}}\varpi)\mathbf{q}_v + \frac{1}{2}\mathbf{Q}_v\mathbf{f} \qquad \mathbf{B}_2 = \frac{1}{2}\mathbf{Q}_v\mathbf{I}^{-1}$$

$$g(\mathbf{x},\dot{\mathbf{x}}) = \begin{bmatrix} \mathbf{A}_1\mathbf{r} + \mathbf{A}_2\dot{\mathbf{r}} \\ \mathbf{B}_1 \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} \mathbf{I}_{3\times3} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_2 \end{bmatrix}$$

➢ Optimal Trajectory Generation Using Legendre Pseudospectral Method

- The optimal control problem of trajectory generation is transformed into constrained Non-Linear Program (**NLP**) through the pseudospectral method and solved through correspondent numerical algorithm.

➢ Optimal Control Problem

- State equation:

$$\sum_{i=0}^{N} D_{ki}\mathbf{x}_i - \frac{t_f - t_0}{2} F(\mathbf{x}^N(t_k), \mathbf{u}^N(t_k), t_k; t_0, t_f) = 0, k = 0,1,2,\ldots,N$$

- Control constraints: including constraints on control output, path, energy matching, configuration and bound conditions.

- Cost Function: for the demand of relative translational control, AMM optimization problem and control output.

$$J_{trans.} = \min(T_f - T_0)$$

$$J_{rot.} = \min \sum_{i=1}^{N_{sat}} \mathbf{T}_{ci}^T \mathbf{W}_{Ti} \mathbf{T} c_i$$

$$J_{ctl.} = \min \sum_{i=1}^{N_{sat}} \sum_{k=1}^{N} [\boldsymbol{\mu}_i(t_{k+1}) - \boldsymbol{\mu}_i(t_k)]^T \mathbf{W}_{\mu i} [\boldsymbol{\mu}_i(t_{k+1}) - \boldsymbol{\mu}_i(t_k)]$$

➢ Optimal Control Problem

- The general control problem

$$\min J = J_{trans.} + J_{rot.} + J_{ctl.}$$

Subject to:

Equality constraints :

Dynamics: $\begin{cases} v(\tau) = \dot{x}(\tau) \\ \dot{v}(\tau) = g(x, v, \tau) + C(x, \tau)u(\tau) \end{cases}$

Enenrgy matching: $h_1(x(\tau_f), \tau_f) = 0$

Configuration: $h_2(x(\tau_f), \tau_f) = 0$

Initial and terminal conditions: $\begin{cases} h_0(x(\tau_0), \tau_0) = 0 \\ h_f(x(\tau_f), \tau_f) = 0 \end{cases}$

Inequality constraints :

Control output: $g_1(x(\tau), u(\tau), \tau) \geq 0$

Anti-"stuck" conditions: $g_2(x(\tau), u(\tau), \tau) \geq 0$

Collision avoidance: $g_3(x(\tau), u(\tau), \tau) \geq 0$

➢ Numerical Computation Method

- The integrated DFP, is applied. A switch of original DFP and BFGS can reach higher accuracy and faster convergence rate under reduced computation complexity.

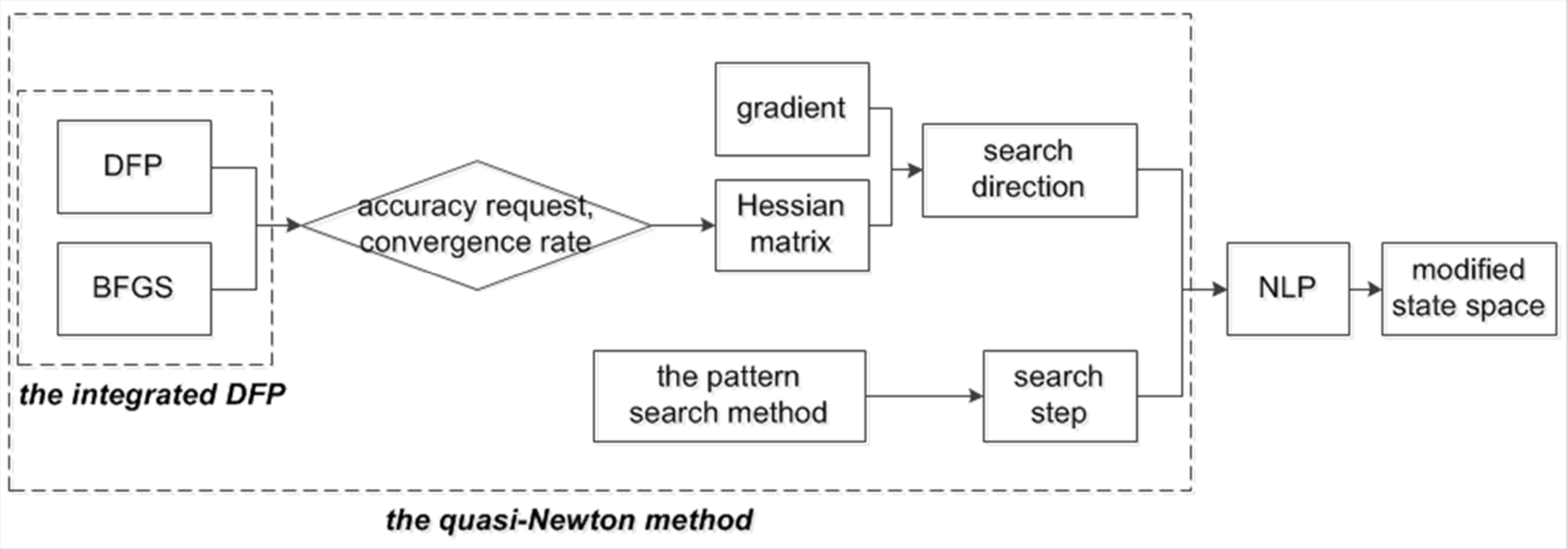


Fig 4. The procedure for solving the NLP

➢ Simulation Results

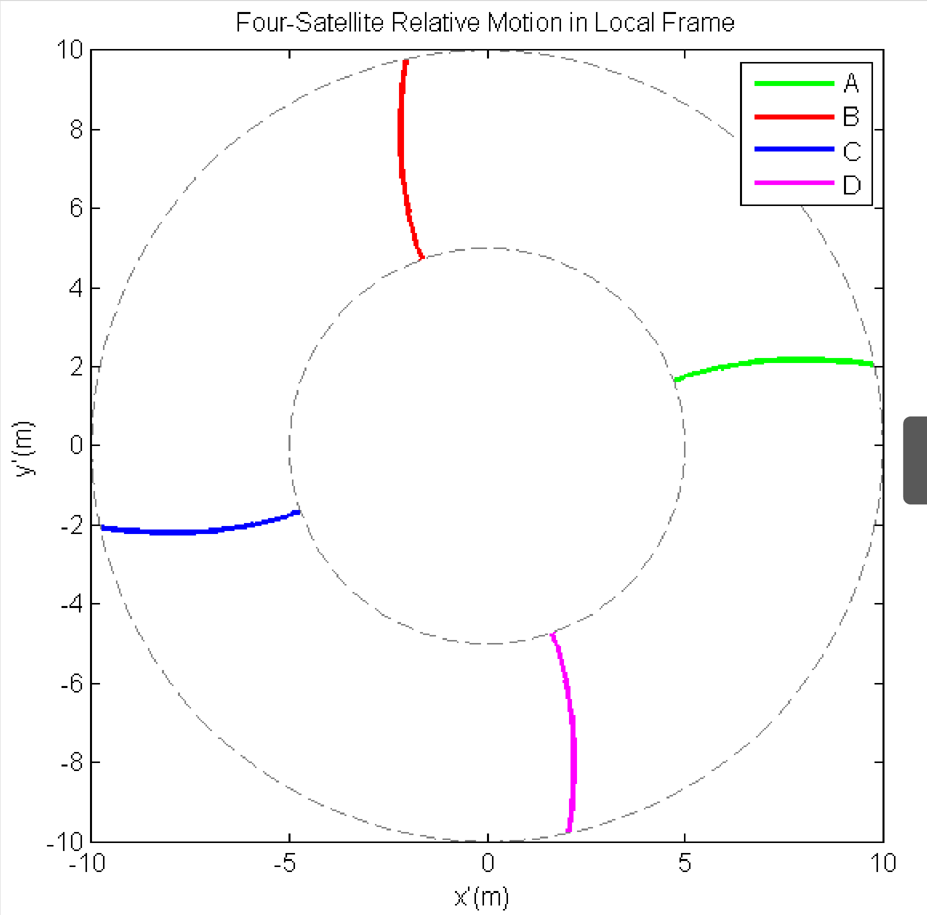
- The formation reconfigures for the purpose of transferring to another relative orbit while keeping the **square** configuration. The initial and terminal phase angles, β_i and β_f, are not fixed in avoidance of extreme control.
- The initial and terminal relative nominal trajectory:

$$\begin{cases} x_i = 5\sin(n_i t + \beta_i) \\ y_i = 10\cos(n_i t + \beta_i) \\ z_i = 5\sqrt{3}\sin(n_i t + \beta_i) \end{cases} \qquad \begin{cases} x_f = 5\sin(n_f t + \beta_f) \\ y_f = 10\cos(n_f t + \beta_f) \\ z_f = 5\sqrt{3}\sin(n_f t + \beta_f) \end{cases}$$

- Parameters:

Mass of satellite (kg)	250	Radius of initial reference orbit (km)	7200
I (kg·m^2)	160	Radius of terminal reference orbit (km)	7200
Radius of initial GCO (m)	10	Maximum magnet moment (H/m)	81250
Radius of terminal GCO (m)	5	Maximum RW torque (N·m)	1

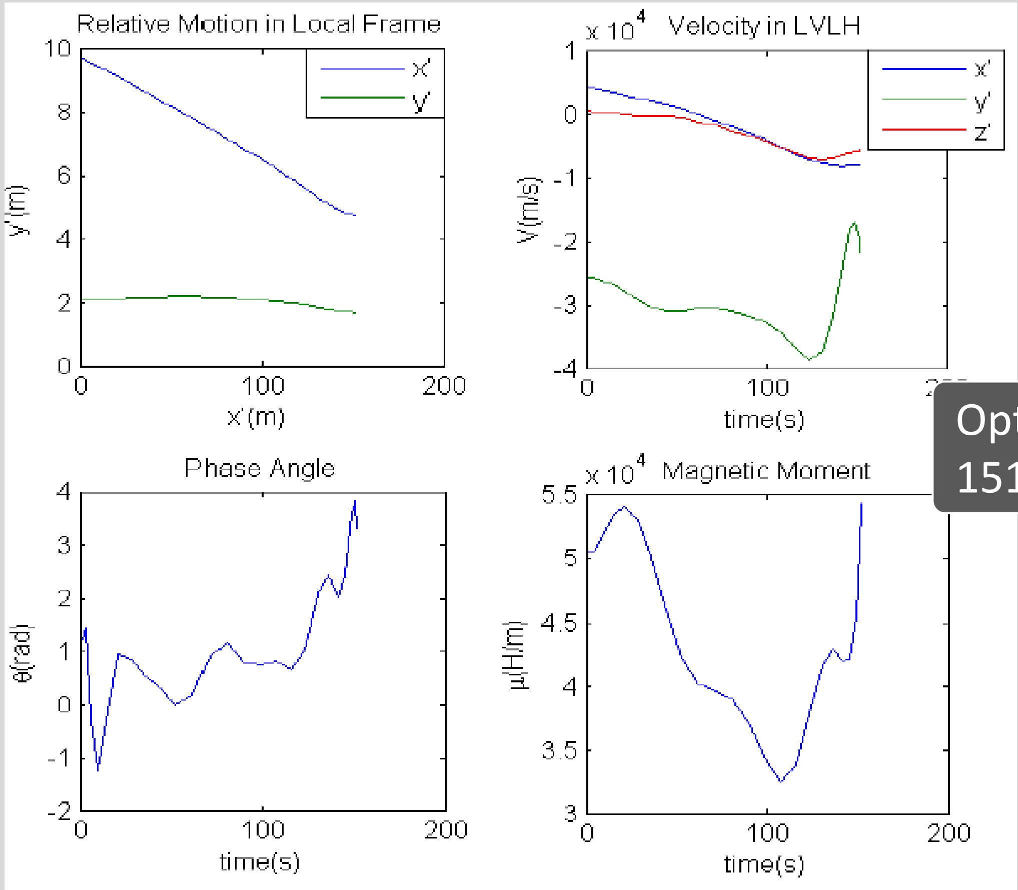
➢ Simulation Results



$r_i=10, \quad r_f=5$

Fig 6. Relative motion of four-satellite planar reconfiguration

➢ Simulation Results



Optimal maneuver time: 151.78 seconds.

Fig 5. State and control results of satellite A

➢ Future work

- The simulation results indicate that the initial and terminal conditions will affect the **convergence rate** and selection of parameters can influence the precision of simulation.
- A series of algorithm with high fidelity and fast convergence rate can be applied to similar optimal control problem.

Thanks for your attention !

Any question can be forwarded directly into

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