OPTIMAL CONTROL OF 6-DOF ELECTROMAGNETIC FORMATION USING THE LEGENDRE PSEUDOSPECTRAL METHOD

Jing Chen, Xiaokui Yue

National Key Laboratory of Aerospace Flight Dynamics, Northwestern Polytechnical University
<table>
<thead>
<tr>
<th>1. Introduction</th>
<th>2. EM Dynamics</th>
<th>3. 6-DOF Dynamics</th>
<th>4. LPM</th>
<th>5. Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1 Introduction</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>2 EM Dynamics</strong></td>
<td><strong>Far-field Model</strong></td>
<td><strong>Four-Satellite Planar Formation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>3 6-DOF Dynamics</strong></td>
<td><strong>Relative Translational Dynamics</strong></td>
<td><strong>Relative Rotational Dynamics</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>4 Legendre Pseudospectral Method (LPM)</strong></td>
<td><strong>Optimal Control Problem</strong></td>
<td><strong>Numerical Computation Method</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>5 Simulation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*National Key Laboratory of Aerospace Flight Dynamics, Northwestern Polytechnical University*
 Formation flying is an enabling technology. Electromagnetic Formation Flying (EMFF) uses HTS coils to provide forces and torque, in exchange of a highly coupled and nonlinear dynamics model.

- The coupling effects:
  - Both magnitude and orientation of the Electromagnetic (EM) force is determined by magnetic dipole strength and relative DOF of the array.
  - When a shear EM force acts, a shear torque is introduced.
Far-Field Model

- **EM force**

\[
F_{ij} = \frac{3\mu_0}{4\pi} \left[ -\frac{5(\mu_i \cdot r_{ij})(\mu_j \cdot r_{ij})}{r_{ij}^7} \right] + \left( \frac{\mu_i \cdot r_{ij}}{r_{ij}^5} \right) \frac{\mu_j}{r_{ij}^5} + \left( \frac{\mu_j \cdot r_{ij}}{r_{ij}^5} \right) \frac{\mu_i}{r_{ij}^5} + \left( \frac{\mu_i \cdot \mu_j}{r_{ij}^5} \right) r_{ij}
\]

\[
F_{ji} = -F_{ij}
\]

- **EM torque**

\[
T_{eij} = \frac{\mu_0 \mu_i}{4\pi} \times \left[ \frac{3r_{ij}(\mu_j \cdot r_{ij})}{r_{ij}^5} - \frac{\mu_j}{r_{ij}^3} \right]
\]

\[
T_{eji} = \frac{\mu_0 \mu_j}{4\pi} \times \left[ \frac{3r_{ij}(\mu_i \cdot r_{ij})}{r_{ij}^5} - \frac{\mu_i}{r_{ij}^3} \right]
\]
Four-Satellite Planar Formation

- Assuming that the distribution of the four-satellite planar formation on the General Circular Orbit (GCO) is symmetrical with respect to the origin of the LVLH coordinate.

- Assuming that the equivalent magnet moment and maneuver trajectories are rotational symmetrical, satellites under similar dynamical circumstance are accordant to each other and can be handled in chorus.
Four-Satellite Planar Formation

\[ F_A = F_A(\mu_A, r_A, \alpha_A, \theta_A) \]

- \( \mu_A \): the dipole strength;
- \( \alpha_A \): the deflection angle of dipole moment in the local frame;
- \( r_A \): the radius of transition orbit; \( \theta_A \): the phase angle in GCO.

Fig 3. Ideal scheme for relative orbit transfer
Relative Translational Dynamics

\[ \ddot{r} = A_1 r + A_2 \dot{r} + a \]

where

\[
A_1 = \begin{bmatrix}
\dot{\phi}^2 + 2 \frac{\mu}{r_1^3} & \dot{\phi}^2 & 0 \\
\dot{\phi} & \dot{\phi}^2 - \frac{\mu}{r_1^3} & 0 \\
0 & 0 & \frac{\mu}{r_1^3}
\end{bmatrix}
\]

\[
A_2 = \begin{bmatrix}
0 & 2\dot{\phi} & 0 \\
-2\dot{\phi} & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

- Both state and control vector need to be normalized to guarantee same magnitude.
Relative Rotational Dynamics

• Describing it in the $i^{th}$ deputy satellite’s body-fixed frame.

\[
\begin{bmatrix}
\ddot{q}_0 \\
\ddot{q}_v
\end{bmatrix} = \begin{bmatrix}
-\frac{1}{4}(\omega^T\omega)q_0 - \frac{1}{2}q_v^Tf - \frac{1}{2}q_v^T\Gamma^{-1}T_A \\
-\frac{1}{4}(\omega^T\omega)q_v + \frac{1}{2}Q_v f + \frac{1}{2}Q_v^T\Gamma^{-1}T_A
\end{bmatrix}
\]

where $T_A = (T_{ei} + T_{ci}) - IA_{ic} \Gamma^{-1}(T_{ec} + T_{cc})$, defined as the equivalent combined control moment. It contains EM torque and RWs moment acted on both satellites and can be optimized to allocate the angular momentum.

• The coupling among EMFF is reflected implicitly in the connection between EM force and torque.
6-DOF Relative Dynamics Model

- Selecting \( x = [r^T, q_v^T]^T \) and \( u = [a^T, T_{\Delta}^T]^T \) as state and control variables.

\[
\ddot{x} = g(x, \dot{x}) + Cu
\]

\[
B_1 = -\frac{1}{4}(\omega^T \omega)q_v + \frac{1}{2}Q_vf \\
B_2 = \frac{1}{2}Q_v I^{-1}
\]

\[
g(x, \dot{x}) = \begin{bmatrix} A_1r + A_2\dot{r} \\ B_1 \end{bmatrix} \\
C = \begin{bmatrix} I_{3\times3} & 0 \\ 0 & B_2 \end{bmatrix}
\]
Optimal Trajectory Generation Using Legendre Pseudospectral Method

- The optimal control problem of trajectory generation is transformed into constrained Non-Linear Program (NLP) through the pseudospectral method and solved through correspondent numerical algorithm.
**Optimal Control Problem**

- **State equation:**
  \[
  \sum_{i=0}^{N} D_{ki} x_i - \frac{t_f - t_0}{2} F(x^N(t_k), u^N(t_k), t_k; t_0, t_f) = 0, k = 0, 1, 2, \ldots, N
  \]

- **Control constraints:** including constraints on control output, path, energy matching, configuration and bound conditions.

- **Cost Function:** for the demand of relative translational control, AMM optimization problem and control output.
  \[
  J_{\text{trans.}} = \min(T_f - T_0)
  \]
  \[
  J_{\text{rot.}} = \min \sum_{i=1}^{N_{\text{sat}}} T^T c_i W_{T_i} T c_i
  \]
  \[
  J_{\text{ctl.}} = \min \sum_{i=1}^{N_{\text{sat}}} \sum_{k=1}^{N} [\mu_i(t_{k+1}) - \mu_i(t_k)]^T W_{\mu_i} [\mu_i(t_{k+1}) - \mu_i(t_k)]
  \]
Optimal Control Problem

- The general control problem

\[ \min J = J_{\text{trans.}} + J_{\text{rot.}} + J_{\text{cyl.}} \]

Subject to:

Equality constraints:

Dynamics:

\[ \begin{cases}
  v(\tau) = \dot{x}(\tau) \\
  \dot{v}(\tau) = g(x, v, \tau) + C(x, \tau)u(\tau)
\end{cases} \]

Energy matching:

\[ h_1(x(\tau_f), \tau_f) = 0 \]

Configuration:

\[ h_2(x(\tau_f), \tau_f) = 0 \]

Initial and terminal conditions:

\[ \begin{cases}
  h_0(x(\tau_0), \tau_0) = 0 \\
  h_f(x(\tau_f), \tau_f) = 0
\end{cases} \]

Inequality constraints:

Control output:

\[ g_1(x(\tau), u(\tau), \tau) \geq 0 \]

Anti-"stuck" conditions:

\[ g_2(x(\tau), u(\tau), \tau) \geq 0 \]

Collision avoidance:

\[ g_3(x(\tau), u(\tau), \tau) \geq 0 \]
Numerical Computation Method
• The integrated DFP, is applied. A switch of original DFP and BFGS can reach higher accuracy and faster convergence rate under reduced computation complexity.

Fig 4. The procedure for solving the NLP
Simulated Results

- The formation reconfigures for the purpose of transferring to another relative orbit while keeping the square configuration. The initial and terminal phase angles, $\beta_i$ and $\beta_f$, are not fixed in avoidance of extreme control.

- The initial and terminal relative nominal trajectory:

\[
\begin{align*}
  x_i &= 5\sin(n_i t + \beta_i) \\
  y_i &= 10\cos(n_i t + \beta_i) \\
  z_i &= 5\sqrt{3}\sin(n_i t + \beta_i) \\
  x_f &= 5\sin(n_f t + \beta_f) \\
  y_f &= 10\cos(n_f t + \beta_f) \\
  z_f &= 5\sqrt{3}\sin(n_f t + \beta_f)
\end{align*}
\]

- Parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of satellite (kg)</td>
<td>250</td>
</tr>
<tr>
<td>Radius of initial reference orbit (km)</td>
<td>7200</td>
</tr>
<tr>
<td>$I$ (kg·m²)</td>
<td>160</td>
</tr>
<tr>
<td>Radius of terminal reference orbit (km)</td>
<td>7200</td>
</tr>
<tr>
<td>Radius of initial GCO (m)</td>
<td>10</td>
</tr>
<tr>
<td>Maximum magnet moment (H/m)</td>
<td>81250</td>
</tr>
<tr>
<td>Radius of terminal GCO (m)</td>
<td>5</td>
</tr>
<tr>
<td>Maximum RW torque (N·m)</td>
<td>1</td>
</tr>
</tbody>
</table>
Fig 6. Relative motion of four-satellite planar reconfiguration

r_i=10, r_f=5
Simulation Results

Optimal maneuver time: 151.78 seconds.

Fig 5. State and control results of satellite A
Future work

• The simulation results indicate that the initial and terminal conditions will affect the convergence rate and selection of parameters can influence the precision of simulation.

• A series of algorithm with high fidelity and fast convergence rate can be applied to similar optimal control problem.
Thanks for your attention!

Any question can be forwarded directly into Chenjing@mail.nwpu.edu.cn