Periodic Relative Orbits of Two Spacecraft Subject to Differential Gravity and Coulomb Forces

Presented by Daan Stevenson for

Drew R. Jones (author) & Hanspeter Schaub (co-author)

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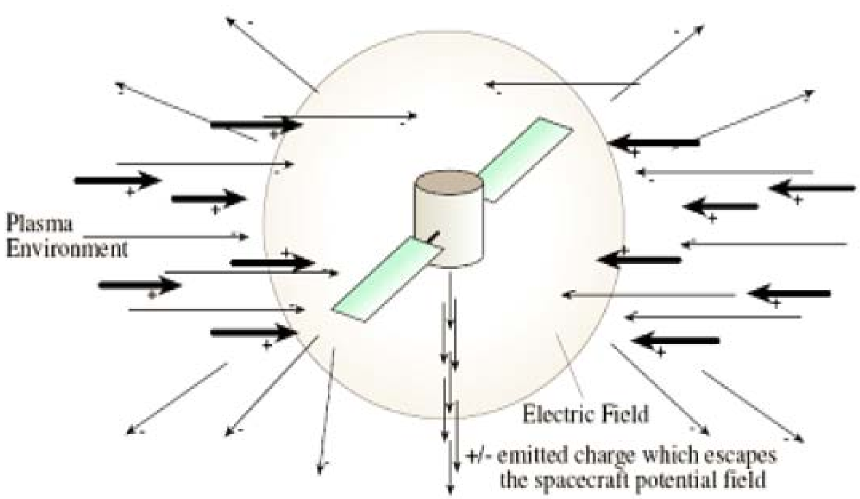
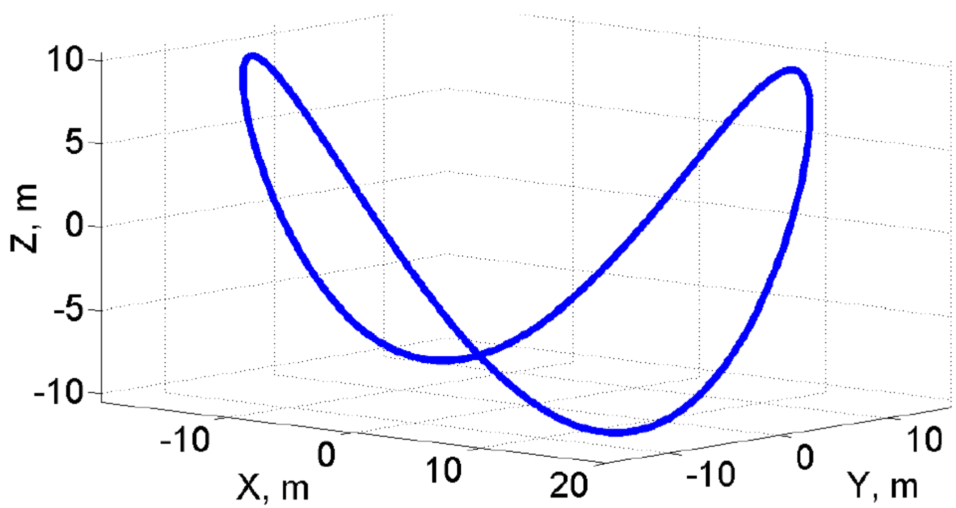
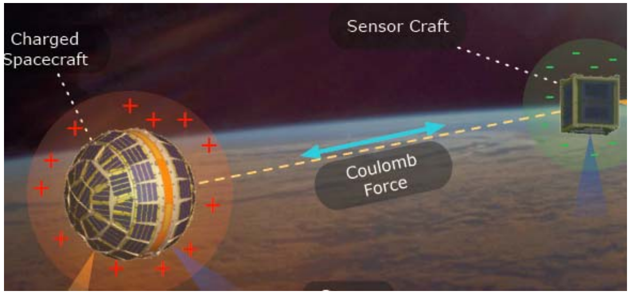


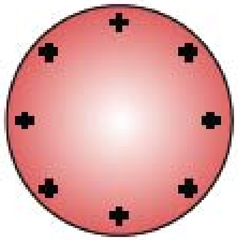


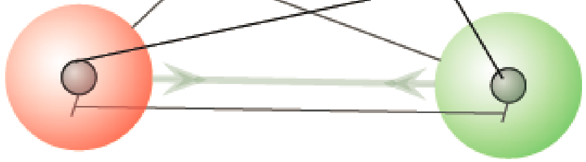
Coulomb Formation Motivation

Coulomb Formation: Close-flying charged craft using electrostatic forces

- Very efficient (ISP~10^{13} s)
- Forces have limited reachability
- Forces introduce NL coupling
- Forces complex to model in the plasma





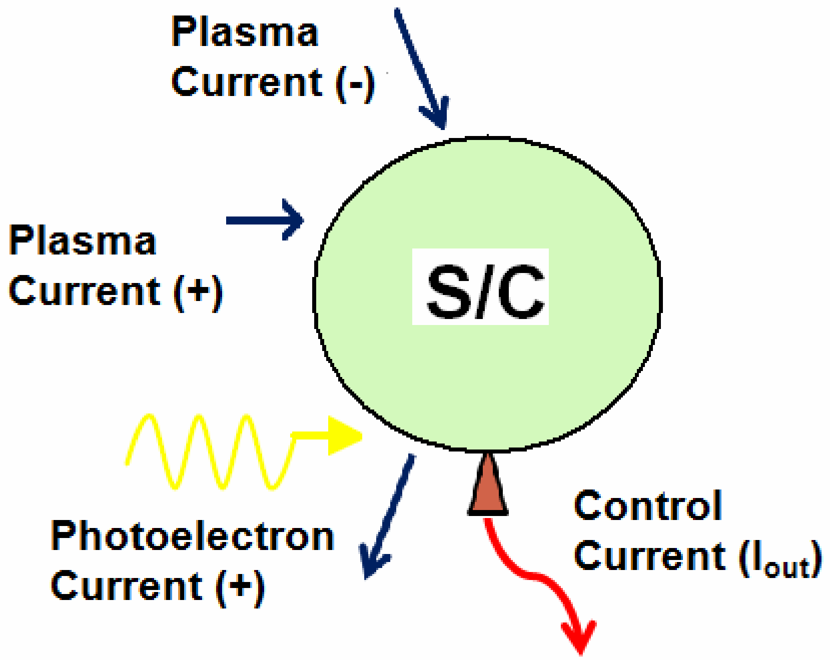


$$F = k_c \frac{q_1 q_2}{r^2}$$

Outline

Spacecraft Charging

- Spacecraft naturally assume non-zero potential (charge)
- Artificially altering potential has been demonstrated
- Spherical s/c model adopted
  - Somewhat an abstraction
  - Enables 1st order calculations and assumptions



Electrostatic Model

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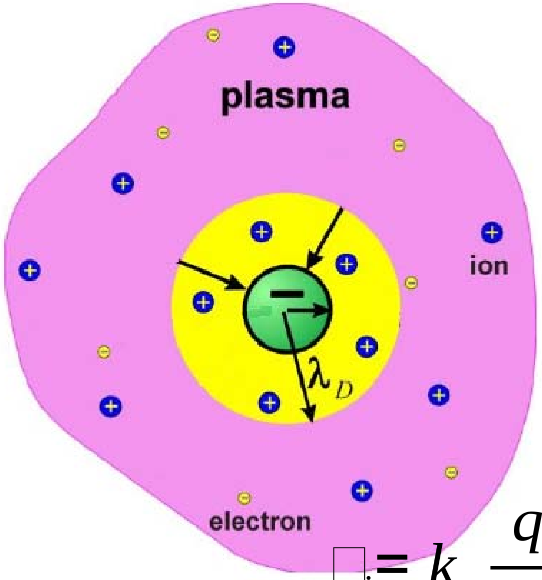
- Spacecraft are not point charges in a vacuum…
  - Adopt approximate and analytical Coulomb force model
  - But still account for plasma shielding effect
  - Assume formations near GEO

Coulomb Force: Point Charges in Vacuum

$$F_{12} = k_c \frac{q_1 q_2}{r_{12}^2}$$



: Finite Bodies with
sma Shielding

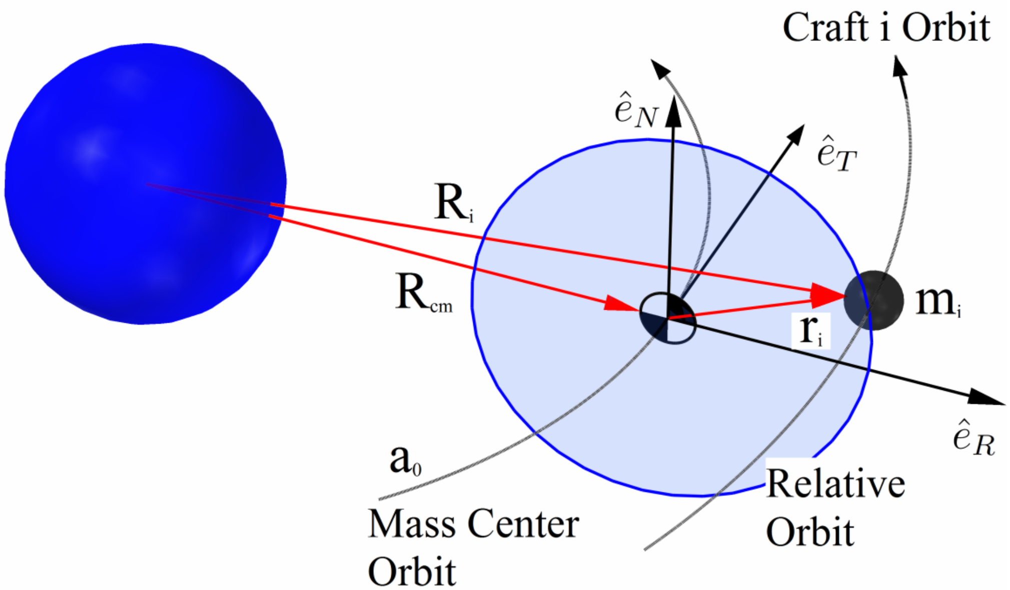
$$\frac{{}_1 q_2 (1 + r_{12}/\lambda_d)}{r_{12}^2 \exp[r_{12}/\lambda_d]}$$

$$\square_i = k_c \frac{q_i}{R_{sc}}$$

Dynamical Model

- Hill Frame Model
- Clohessy-Wiltshire relative motion EOM

$$\sum_i m_i \mathbf{r}_i = 0$$



- With Debye-Hückel Coulomb force model:

$$\ddot{\mathbf{r}}_i = \begin{bmatrix} 2\omega\dot{y}_i + 3\omega^2 x_i \\ -2\omega\dot{x}_i \\ -\omega^2 z_i \end{bmatrix} + \left[\frac{k_c\ q_i}{m_i}\sum_{j\neq i}\frac{q_j\ e^{-r_{ij}/\lambda_d}}{r_{ij}^3}\left(1+\frac{r_{ij}}{\lambda_d}\right)\mathbf{r_{ij}}\right]$$

Reduced Dynamics

- Dynamics normalized, one craft removed
  - s/c 2 motion is always dependent, via CM constraint
  - Scaled charge products introduce time transformation

$$Q_{12} = q_1 q_2$$

$$\tilde{Q}_{12} = \frac{k_c \, Q_{12}}{\omega^2} \qquad \mathrm{d}\tau = \omega \mathrm{d}t \qquad (\zeta)' = \frac{\mathrm{d}\zeta}{\mathrm{d}\tau} = \frac{1}{\omega}\frac{\mathrm{d}\zeta}{\mathrm{d}t}$$

- Scalar constant of motion is shown to exist
  - Holds for N-craft Hill frame systems with internal forcing
  - For two-craft formation, the motion constant is:

$$x(\tau)y''(\tau) - y(\tau)x''(\tau) = 0$$

Dynamical System Analysis

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- Seek periodic solutions to vector ODE $\mathbf{X}' = \mathbf{F}(\mathbf{X},\ \mathbf{u},\ \tau)$
  - Dynamics linearized about such solutions

$$\delta\mathbf{X}'(\tau) = \left(\frac{\partial\mathbf{F}}{\partial\mathbf{X}}\right)\bigg|_{(\mathbf{X}^*,\mathbf{u}^*)} \delta\mathbf{X}(\tau) \ = \ \mathbf{A}(\tau)\ \delta\mathbf{X}(\tau)$$

$$\delta\mathbf{X}(\tau) = \mathbf{\Phi}(\tau,0)\ \delta\mathbf{X}(0)$$

$$\mathbf{\Phi}'(\tau,0) = \mathbf{A}(\tau)\ \mathbf{\Phi}(\tau,0)$$

  - STM matrix has identical symplectic property as known for linear dynamics about libration points in CRTBP
  - Asymptotically stable solutions cannot exist

$$\mathbf{J}\mathbf{A}^T = -\mathbf{A}\mathbf{J} \qquad \mathbf{\Phi}\mathbf{J}\mathbf{\Phi}^T = \mathbf{J} \qquad \mathbf{J} = \left[\begin{array}{c|c} \mathbf{0} & \mathbf{I} \\ \hline -\mathbf{I} & \mathbf{G} \end{array}\right]$$

1. $\det(\mathbf{\Phi}) = |\mathbf{\Phi}| = 1$
2. At least one Floquet multiplier has modulus of unity: $|\sigma_i| = 1$
3. The σ_i appear in reciprocal pairs (i.e. if σ_i is eigenvalue, then so is $\sigma_j = 1/\sigma_i$)

Periodic Coulomb Formations

Periodic Coulomb Formations

- Assume positions as simple harmonic oscillators
  - Solve time-varying charges that produce assumed motion
  - Motions are restricted (cannot be assumed arbitrarily)

- Relative instability measured via Floquet multipliers

- Dynamical coupling permits 3 solution families
  - Orbit-Normal oscillations only (1D)
  - Reference orbit-plane (2D) motions
  - Full state (3D) motions

Reference Orbit Plane Solutions

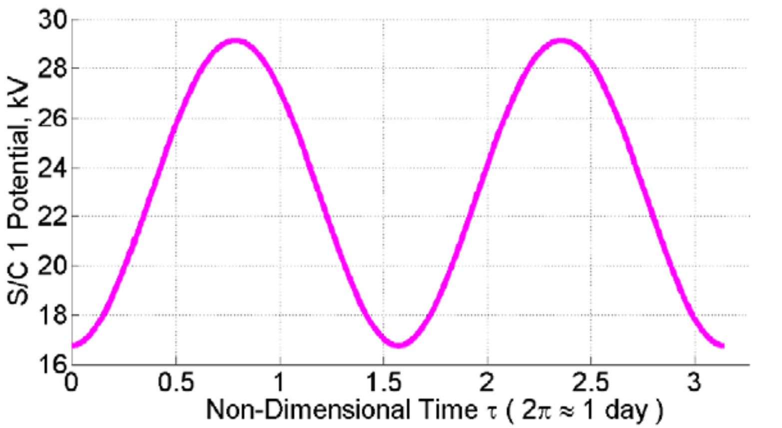
$$x(\tau) = A_x \cos(\theta_x \tau) \qquad y(\tau) = A_y \sin(\theta_y \tau)$$

1) $x(\tau)y''(\tau) - y(\tau)x''(\tau) = 0$ → $\theta_x = \theta_y$

2) Dynamical Coupling (Coulomb terms): → $\left(\frac{A_y}{A_x}\right) = \frac{-3 \pm \sqrt{9 + 16\theta^2}}{4\theta}$

- Two roots of quadratic yield two families (cases A/B)
  - Relative orbit is ellipse
  - Major axis along either radial (case A) or transverse (case B)
  - Short period (case B) orbits → least unstable

$$\tilde{Q}(r(\tau)) = \frac{-1}{\Psi(r)}\left[\theta^2 + 3 + \left(\frac{-3 \pm \sqrt{9 + 16\theta^2}}{2}\right)\right]$$



(b) Case B: $A_x < A_y$

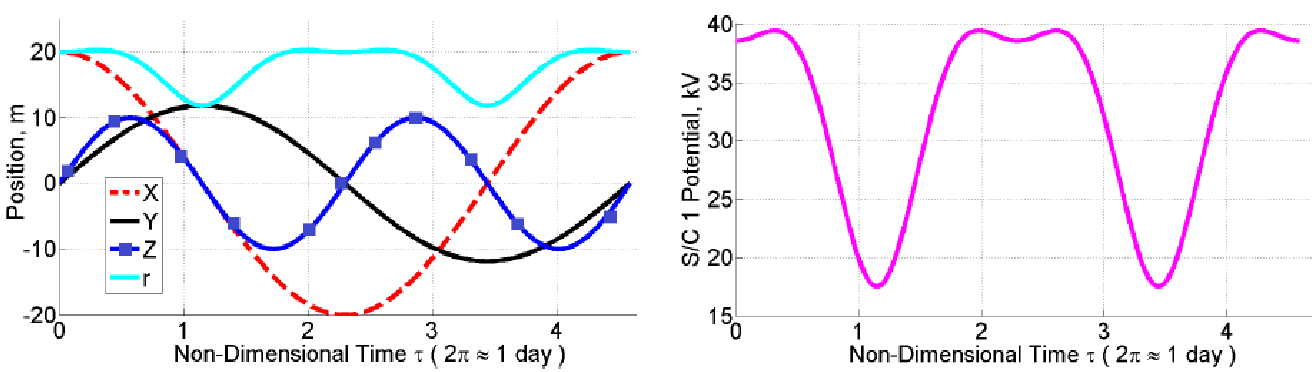
Full State Solutions

- Same in-plane assumed functions (same restrictions)
- Additionally $z(\tau) = A_z \sin(B_z \theta \tau)$

1) Out of plane frequency factor must be even integer $\longrightarrow$ $B_z = 2, 4, 8 \ldots$

2) Time period no longer free, must satisfy: $\longrightarrow$ $8\theta^2 + \left(-3 \pm \sqrt{9 + 16\theta^2}\right)\left[\theta^2(1 - B_z^2) + 1\right] = 0$

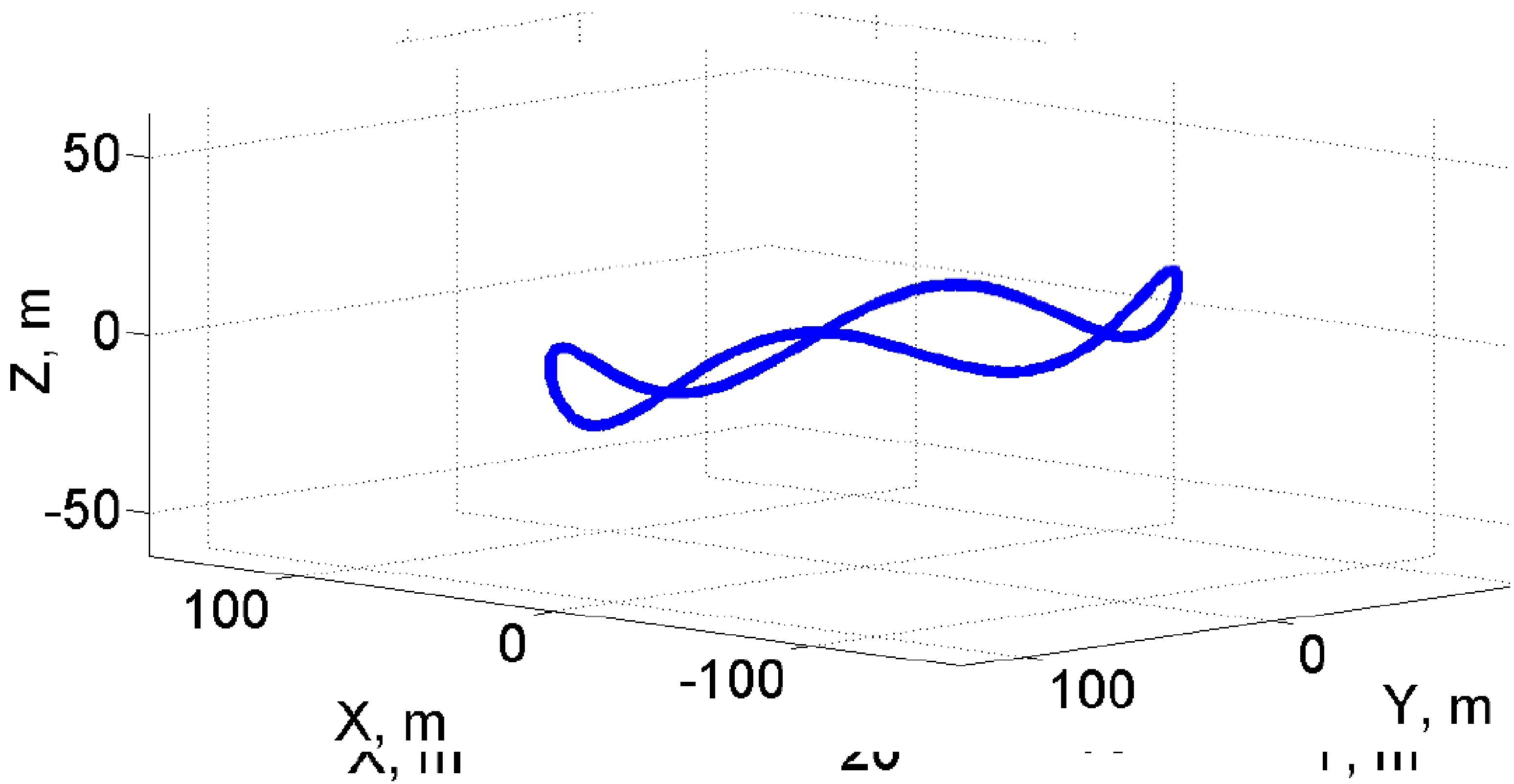
  - Projected trajectory is ellipse aligned as in case A/B
  - Case B orbits have smaller max. Floquet multiplier



(a) S/C 1 Position History (b) S/C 1 Potential History

Periodic Coulomb Formations

- Example relative orbit
  - S/C 1 Trajectory → like saddle, x-y projection is ellipse



Full State Periodic Solution: Case B, $B_z=4$

Circumnavigating 2-s/c Orbit

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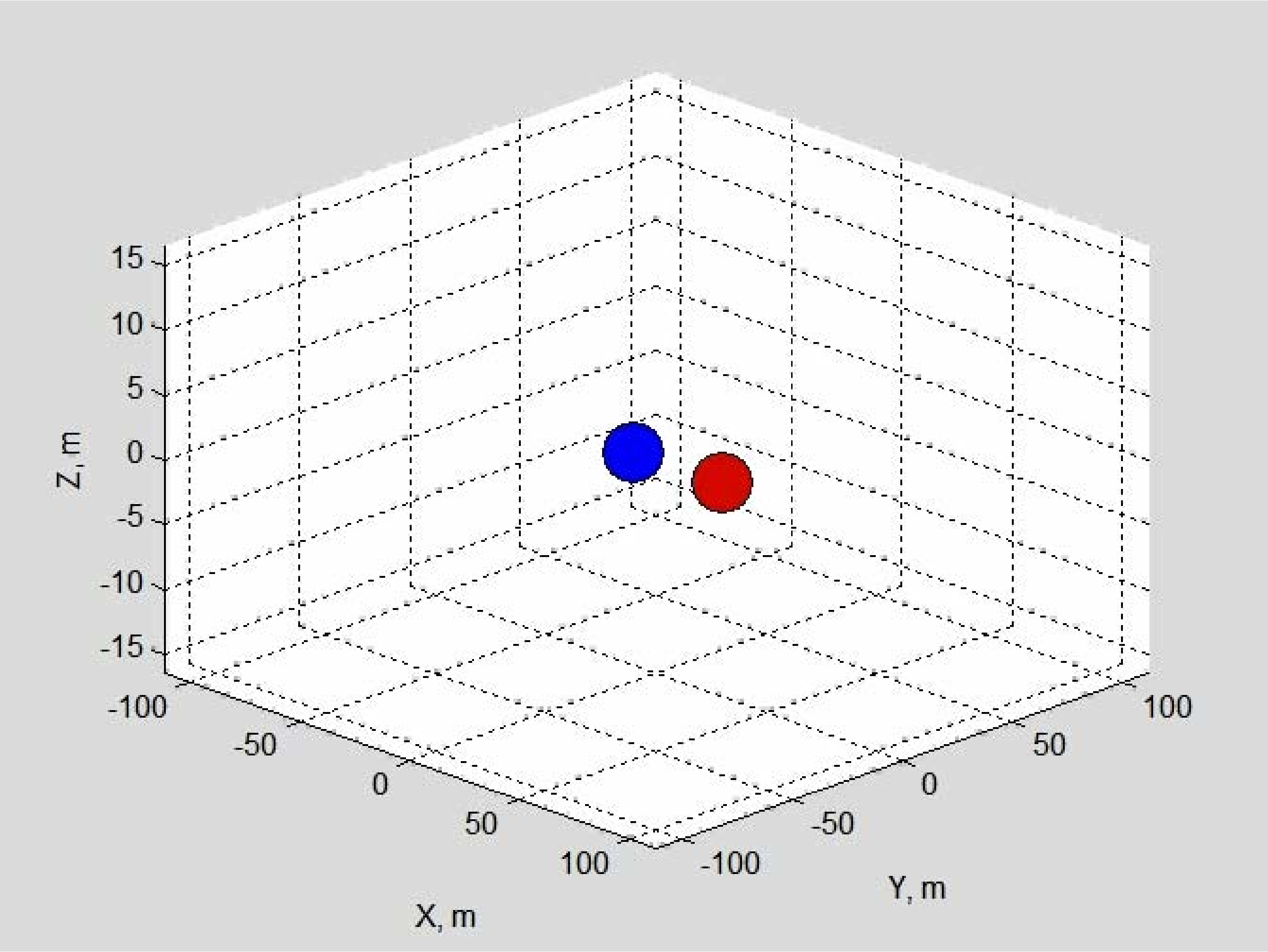
- Another Example
  - Both craft trajectories shown, different mass vehicles



Perturbed Periodic Coulomb Formation Solutions Propagated in Inertial Frame

Perturbed Dynamical Model

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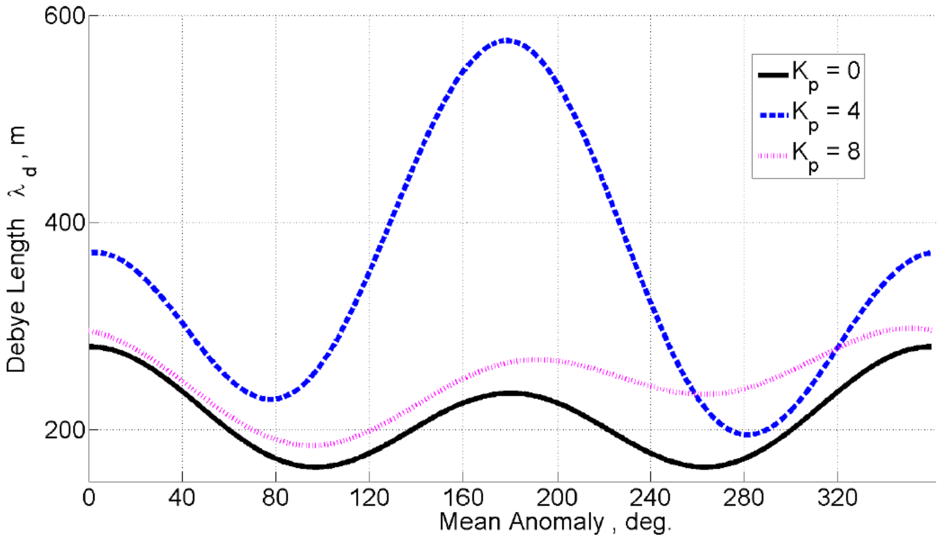
- Accuracy of solutions in higher fidelity model
  - Revert back to Newtonian gravity, inertial frame
  - Transformations between ECI and Hill frames
  - Include primary perturbations

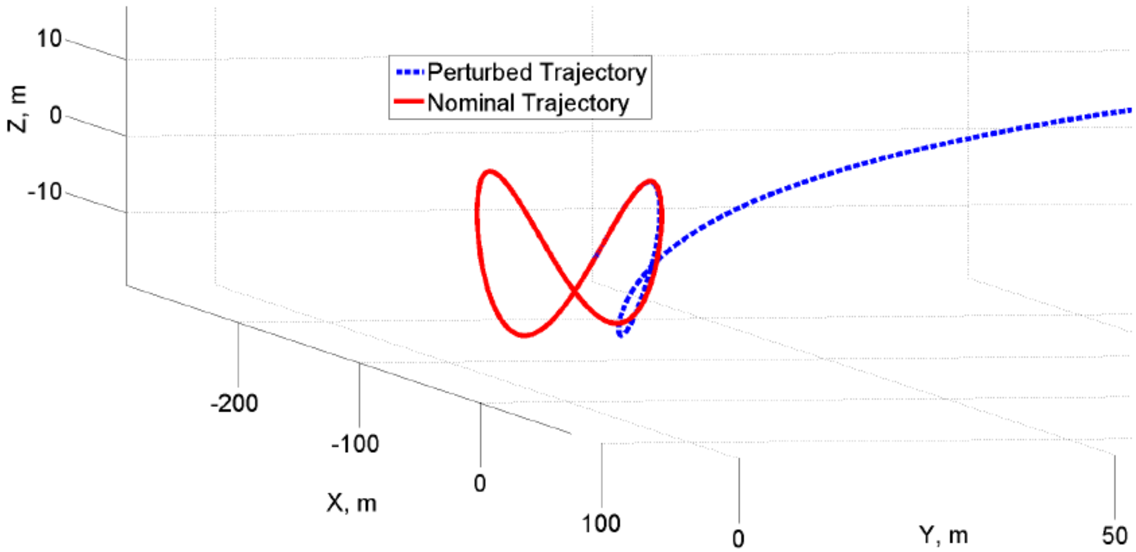
Solar Radiation Pressure using "Cannonball Model"

$$f_{\text{srp}} = C_R \frac{\pi R_{sc}^2 \; \Theta}{m_i \; c}$$

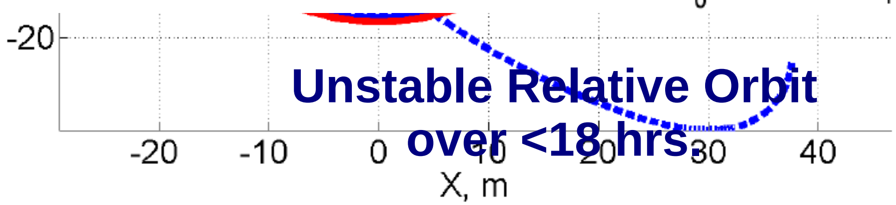
Induced Perturbation due to Parametric Uncertainty (Model Error) in Debye Length



Perturbed Periodic Solutions

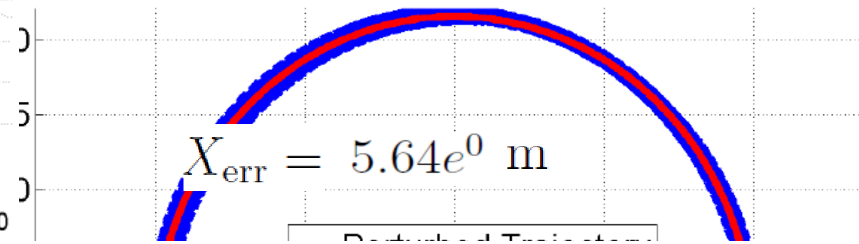


Unstable Relative Orbit over 10 Revolutions

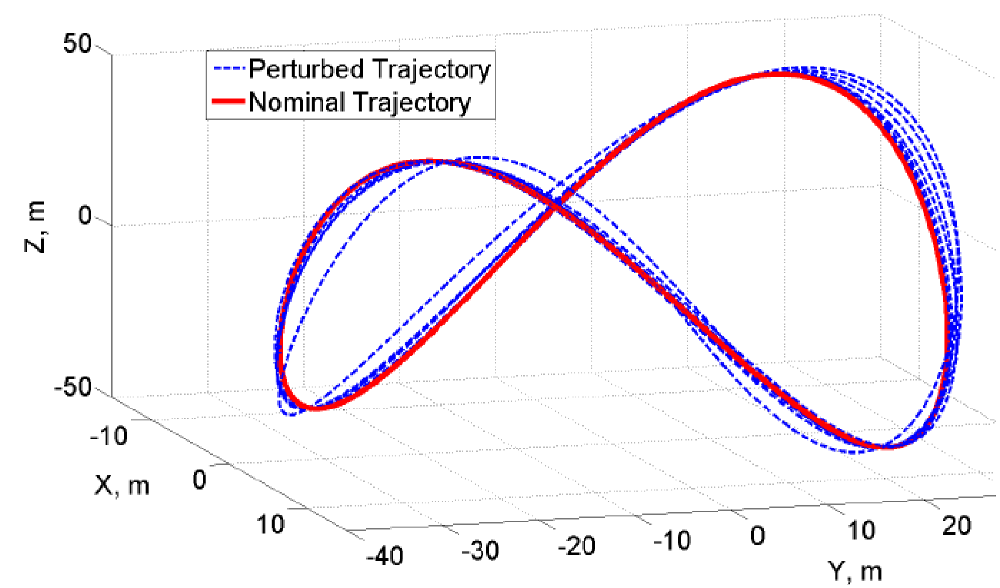


Unstable Relative Orbit over <18 hrs.

$$X_{\text{err}} = 5.75e^{-2}\ \text{m}$$



$$X_{\text{err}} = 5.64e^{0}\ \text{m}$$



Stability over 40 Revolutions

Relative Orbit Exhibiting Marginal Stability over ~300 hrs.

Conclusions

- Application of new theory opens multiple pathways
  - The existence of periodic solutions enabled by open-loop electrostatic forcing
  - Stability properties assessed via Floquet theory
  - Some periodic solutions are very nearly stable
- Future Work
  - Other admissible periodic solutions (represent w/ finite Fourier)
  - Feedback control of periodic solutions
  - Test these solutions (and derive new ones) with relaxed assumptions and higher order Coulomb force modeling
  - Precisely measure, control, and estimate s/c potentials (and Coulomb forces) in order to realize these motions in practice?

QUESTIONS

Please refer further questions to Drew Jones: drjones604@gmail.com

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EXTRA SLIDES

Electrostatic Model Assumptions

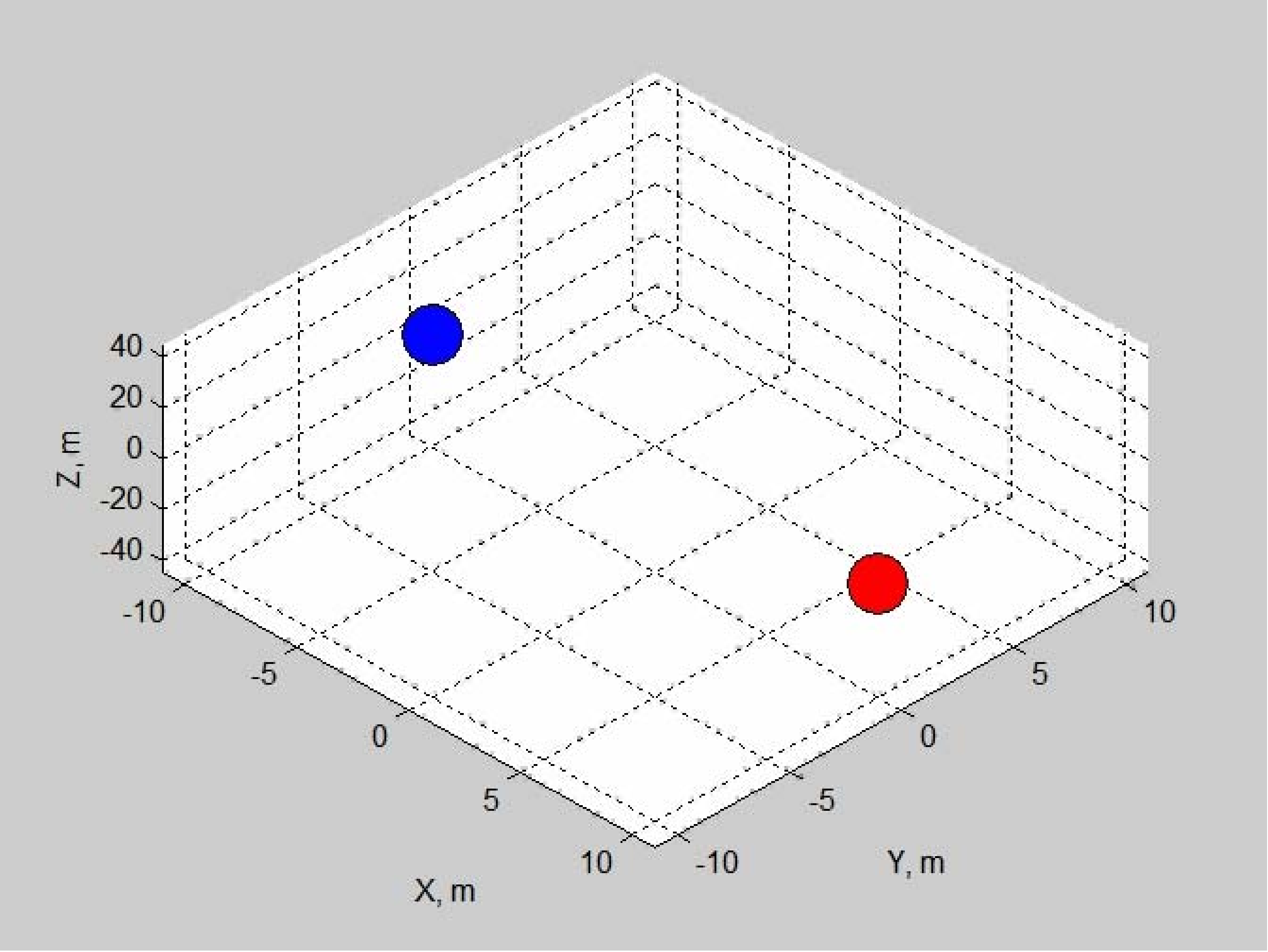
- True force between finite s/c: solution to time-dependent Vlaslov-Poisson PDE

$$F \approx k_c \frac{q_1 q_2 (1 + r_{12}/\lambda_d)}{r_{12}^2 \exp[r_{12}/\lambda_d]} \qquad \square_i = k_c \frac{q_i}{R_{sc}}$$

- Assumptions for approximate model accuracy:
  - Craft act equivalently as spheres at a distance
  - Conservative plasma shielding
  - Decoupled capacitances
  - Constant and finite λ_d (at GEO order of 100 m)
  - Formations near GEO (λ_d negligible over R_{sc})
  - Lower bound on separation distance: $r_{12} > 10\ R_{sc}$

Periodic Coulomb Formation

- Case A, $B_z = 2$, $A_x = 10$ m, $A_y = 6$ m, $A_z = 40$ m (17 hrs)



Periodic Coulomb Formation

- Case B, $B_z = 4$, $A_x = 20$ m, $A_y = 125$ m, $A_z = 5$ m (4 days)