FORMATION FLYING MISSION WITH SIMPLIFIED GUIDANCE

M.P.RAMACHANDRAN

Scientist/Engineer, Flight Dynamics Group, ISRO Satellite Centre,
Airport Road, Bangalore 560 017, India,
91-80-2508 4409, mprama@isac.gov.in

Abstract: *Formation flying is described as harnessing and exploring the dynamics of relative motion of satellites. Relative motion is brought in this paper for low earth orbiting satellites. This accommodates oblate and effects of other perturbations. Geometric approach is used to capture the secular relative dynamics. A simplified guidance is brought out that is valid for larger separations and maintaining a bounded formation for extended durations. Onboard autonomy is achieved for this duration eliminating ground constraints. The guidance along with impulsive maneuvers once or twice helps to realise orbit control of the formation. The relative motion is illustrated by simulation for a PCO type formation in the presence of all perturbations.*

Keywords: *Relative-dynamics, oblate, secular, guidance, noise*

1. Introduction

Polar Satellite Launch Vehicle (PSLV) has an enviable record in accurately launching multiple satellites in polar orbits. These include small satellites from all over the world with optical, microwave and radar payloads. Formation flying technologies has been identified among ISRO's advanced R&D plans. Here it may be noted that this technology means development of inter-satellite communication, formation guidance and satellite orbit control. The mission scenario is based on the type of payload the satellite carries, the power requirements and data generated onboard the satellite. The satellites are identical in their ballistic coefficients.

Formation flying enhances the science in the presence of baseline observation and also the coverage durations. It brings out a gradual decay in mission fulfillment apart from redundancies. Optimal formation operations are possible by interchanging the satellite positions to enable equal fuel utilization.

Maintaining close formations is not easy from control point of view. Large separations enhance wide baseline for payload measurements. However most the relative dynamics models are valid for linear range that is close formations. Geometric approach [1] is ideal for it does not have any restriction on separation distances.

Formation is either in open loop or in closed loop. Open loop means realizing formation with ground control commands. Whereas the closed loop enables automation and ground intervention is significantly reduced. Automation also eliminates visibility constraints. This mission goal is being addressed in this paper regarding a plan for guidance. Relative dynamics is the guidance and engineering goal, when the satellite formation is in active control.

The Clohessy-Wiltshire (C-W) Hills equations considers relative dynamics when the chief satellite or reference orbit has circular motion in constant earth gravity . This may not be reality for extended periods. For example when the perigee is not arrested, the oblate effect allows the growth in eccentricity. Hence the C-W model has limitations. Besides the oblate earth effect there are other perturbations like the third body attraction such as moon or sun attraction, solar radiation pressure and atmospheric drag effects. These effects are 1000 times weaker than the oblate effect. Yet they contribute a secular variation when considering the oblate effect alone.

Satellites experience all the forces. There are two types of corrections; one due to natural forces and other due to the formation control. Perturbation effects are insignificant for rendezvous for their separation gets closer and the differential effects are minimal. On the other hand for formation flying this is not the case. Controlling the formation by having models that accommodate oblate effect alone can also cause more fuel loss. This is more so when the formation is planned for an extended period. This is the salient aspect that is achieved in this paper.

The outcome enables satellite in realising payload operations with more autonomy and following schedules which are a function of time.

2. Relative Guidance

The relative dynamics model usually resides with the deputy and this maintains the selected formation with respect to the chief satellite. The chief is passive. This logic applies to all the deputies in the formation.

This section is made up of two parts namely (1) Measurements Smoothing and (2) Relative Dynamics

2.1 Measurements Smoothing

Satellite Position System (SPS) is onboard and provides position information with GPS data. The inter-satellite link is used to transfer raw measurements from one satellite to the other.

This has certain noise and the derived velocity is also unreliable. Smoothing usually refines the position and estimates velocity more accurately. Navigation accuracy needs to be one order more accurate to the control box that is decided. That is if relative distances are to be maintained to the accuracies of 300m then the navigation system needs to be accurate to 30m.

Continuous thrust type control constantly modifies the orbit and payload data collection. A filter needs to be comprehensive to accommodate such continuous thrust and that this needs to be onboard. Also continuous maneuver is possible only with low thrusters like ion thrusters where the power requirement is higher.

Purely kinematic nature of estimation correcting latency is more suited for impulsive satellite control. Kinematic orbit determination (OD) does not use any model and relies on the measurements. This data could either be position or phase differences. The later can be very accurate and is possible when phase data is available from the receiver.

In this paper only the position data is available from the receiver. The navigation system is invoked before the maneuver or payload operations. This greatly simplifies the onboard navigation and control.

The position data is used and the batch filter is invoked [2]. The measurements are available at equal intervals of time and 'n' such measurements are used. The spline is used for smoothing purposes and the regularisation by minimizing the functional:

$$\Psi(f) = (1/(n-1)) \sum_{i=2}^{n-1} (\breve{y}(x_i) - f(x_i))^2 + \alpha \, || f'' ||^2 \qquad (1)$$

where $\breve{y}(x_i)$ is the measurement and f represent the spline fit. It has to be noted that the regularisation parameter α is selected apriori based on the noise of the sensor in the position. The regularisation approach ensures a refinement of the position data and enhances the accuracies of the velocity data in the presence of noise.

When we use the position data with accuracies of 30m we arrive from the batch filter a velocity estimate better than 3m/sec. Here the batch filter involves a part of matrix multiplication on the measured positions.

$$f = D \, \breve{y} \qquad (2)$$

Considering the specified epoch we need to consider only 4 rows and N measurements. This involves 12N floating point operations for all the three axes. With an additional 12 more operations we can determine the velocity at a specified epoch. This completes the OD. As mentioned earlier these limited computations, say with n=9, are performed about a minute before the specified times on the orbit. This shall be clearer in the next section.

2.2 Relative Dynamics

Instantaneous orbit is called osculating orbit. This contains short, long and secular effects of the perturbation.

It would not be fruitful to include short and long periods in the formation control. The short and long period of each satellite do not cause unbounded relative motion. These are more relevant in close proximity formations with limited mission life and with appropriate sensors, control systems and have continuous thrust facility. There is a need to consider the secular variations in the mean motion. In fact mean motion is used to plan the imaging schedules.

The smoothed position corresponds to the osculating. It needs a conversion to obtain the mean or averaged motion. Instantaneous mean conversion for linear dynamics that is neglecting higher order eccentricity effects requires additional computational processes. This applies individual satellites. In our paper we explore a possibility of avoiding this additional conversion within reasonable accuracies yet valid only at certain times in every orbit.

Most of the relative motions start with CW equations solutions. This is in closed form with respect to time [1]. The use of time explicit solutions are more suited for satellites while dealing with orbits that are nearly circular. Disadvantage of CW is that it assumes linearity in the baseline separation; that is the separation is small compared to the semi major axis. Subsequent developments have considered reference orbit to have eccentricity [3] and also the oblate effect [4]. These continue the linearity constraint. Here a wealth of models, each having distinct and significant advantages are available and discussed in [1]. The geometric approach suggested in [5] is useful for it accommodates the oblate J_2 effect without any linearity constraint.

Let r1 {a1,e1,i1,Ω1,ω1,M1} and r2 {a2,e2,i2,Ω2,ω2,M2} represent the two satellites. We note that the satellites are from the same launch and hence the inclination i1 is very close to i2 like the semi-major axis.

Before we go further we define the LVLH plane. Here the unit vector x is along radial direction , z is along the angular momentum vector and y completes the triad. A Projected Circular Orbit (PCO) formation is one wherein the projection of the relative orbit is a circle in the y-z plane. A LEO satellite in PCO that is more useful by having a separation at the equator and is illustrated as:

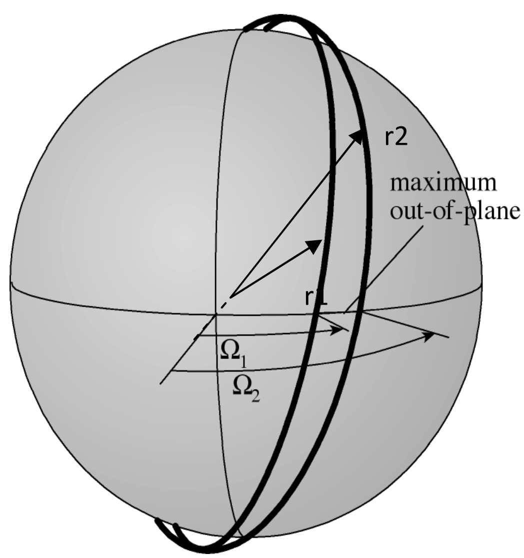

Figure 1. Projected circular formation

The first order secular perturbation that includes the oblate effect on the spherical earth, with equatorial radius R is [6]:

$$
\left.\begin{aligned}
&\omega 1' = (3/2)\, J_2\, (R^2/\, p^2)\, n\{\, (2-\, (5/2)\sin^{2} i1\}\, t \\
&\Omega 1' = -(3/2)\, J_2\, (R^2/\, p^2) n\, (\cos i1)\, t \\
&M1' = \eta\, t \\
&\eta = n\, [\, 1 + (3/2)(J_2\, R^2/p^2)\, (1-(3/2)\, \sin^2 i)(1-e^2)^{/12}] \\
&n = \sqrt{\frac{\mu}{a}}\, (1/a) \\
&p = a(1-e^2)
\end{aligned}\right\} \qquad (3)
$$

Here the a1,e1 and i1 are the momenta elements do not exhibit secular variation over the duration ‘t’. Periodic variations are neglected. This is applied to the second satellite also. Having obtained the orbital elements as a function of time; the relative dynamics expressed in the LVLH frame of the chief is as follows:

$$
\begin{Bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{Bmatrix} = [\, C1\ \ C2^{T} - I] \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} \qquad (4)
$$

Here the matrices C1 and C2 denote the transformation matrix relating the LVLH frame to the inertial frame and I is the identity matrix of size (3x3).

The orthogonal matrix C^T in general is (*c* and *s* denote cosine and sine functions)

$$
\begin{Bmatrix} c\Theta c\Omega - c i s\Omega s\Theta & -c\Omega s\Theta - c i s\Omega c\Theta & s i s\Omega \\ s\Omega c\Theta + c i c\Omega s\Theta & -s\Omega s\Theta + c i c\Omega c\Theta & s i c\Omega \\ s i s\Theta & s i c\Theta & c i \end{Bmatrix} \qquad (5)
$$

After certain mathematical manipulations we can get the actual relative motion as:

$$\left.\begin{aligned} \Delta x &= f_x(i1, i2, \Delta\Omega, \Delta\Theta, \Theta1) \\ \Delta y &= f_y(i1, i2, \Delta\Omega, \Delta\Theta, \Theta1) \\ \Delta z &= f_z(i1, i2\ \Delta\Omega, \Delta\Theta, \Theta2) \end{aligned}\right\} \quad (6)$$

where $\Delta\Omega$ and $\Delta\Theta$ are the differences in the right ascensions of the ascending node of the two satellites and differences in the argument of the latitude of the two satellites. Further we have:

$$\left.\begin{aligned} \delta x &= r2(1+\Delta x) - r1 \\ \delta y &= \Delta y\, r2 \\ \delta z &= \Delta z\, r2 \end{aligned}\right\} \quad (7)$$

where δx,δy and δz are the mean position in the radial, along track and across-track. We have used the secular variation of the oblate effect.

There is a secular variation(apart from the periodic variations) when a full forced model is compared with the model describing the above J_2 effect. Here we propose to accommodate the secular variations alone in each of the satellite for one orbit using:

$$\omega c\ 1 + \Theta c\ 1 = \omega 1 + \Theta 1 + s(t) \quad (8)$$

That is the argument of latitude of the satellite 1 in Eq (3) which is ($\omega1 + \Theta1$), is corrected to match close to the argument of latitude of the satellite from full forced model/exact that is ($\omega c1 + \Theta c1$) to a reasonable accuracy using the correction s(t). This shall become more clear from the simulation in the next section.

We then use the revised argument of latitude and then determine the relative orbit (6) and (7). This correction is realised onboard with minimal ground commands. The principle is to capture the along track secular deviation with respect to satellite experiences all forces. The correction can be kept identical for both the satellites within reasonable accuracies. The other periodic deviations of each satellite relative to other remain periodic and thus can be neglected. This enables the guidance being simplistic with minimal of ground intervention.

Also by this approach at certain points of time in the orbit we have a match in positions that are close to the osculating orbit. It is at these points that the measurements enable relative positions. The correction in the LVLH frame is estimated using guidance and based on the deviations impulsive thrust operations are then carried out.

Out of plane variations have been avoided. Two reasons are (1) Out of plane deviations from $J_{2,2}$ onwards have minimal secular effect over a day and (2) The along track foot-print of any satellite constitutes its payload resolution while across track is mostly is inherent within the area or array sensor shared by the satellites

3. Simulations

The simulations carried out here shall adequately describe the relative dynamics mentioned in the previous section. The satellites have a semi-major axis of 7106.863kms, e= 0.00032 and i= 97.912 degrees. The longitude of the ascending node of Sat 1at the start is 204.997 degrees (differs to Sat 2 by 0.38 degrees) . The j2 motion described by (3) and the exact model [6] are obtained. The plot in Figure 2 describes the deviations in position between the full force model orbit/exact orbit and that orbit which accounts only oblate effect. We notice the secular variations. The spacecraft needs to control this from position and pointing aspects. Fuel is lost.

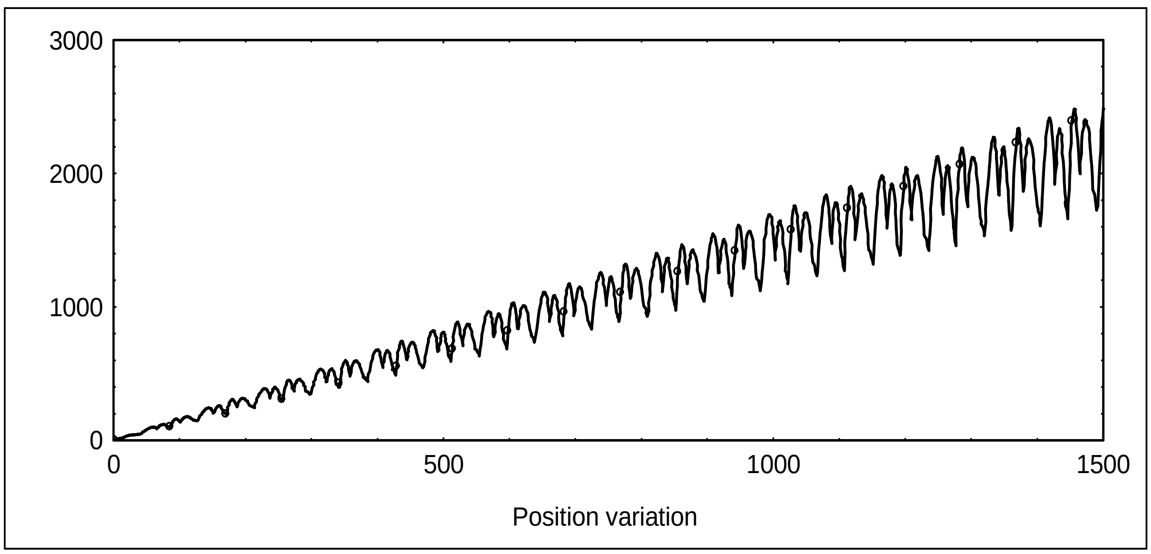

Figure 2: Position variation

The deviation in the argument of latitude between the oblate and full forced model is obtained. Next, a correction s(t) that makes the argument of latitude based on the oblate effect close to the argument of latitude of the full forced model is found. This is carried out for both satellites for one orbit and as the satellites are not very far the corrections is averaged. In this case the correction was

$$s(t) =(0.00125) + (0.000169)t + (-0.000001116)t^2 \qquad (9)$$

This correction is added at the end of each orbital period to the argument of latitude derived from (3).

The figure 3 is the same position difference between full forced model and that which is obtained using Eqs (9) and (8).

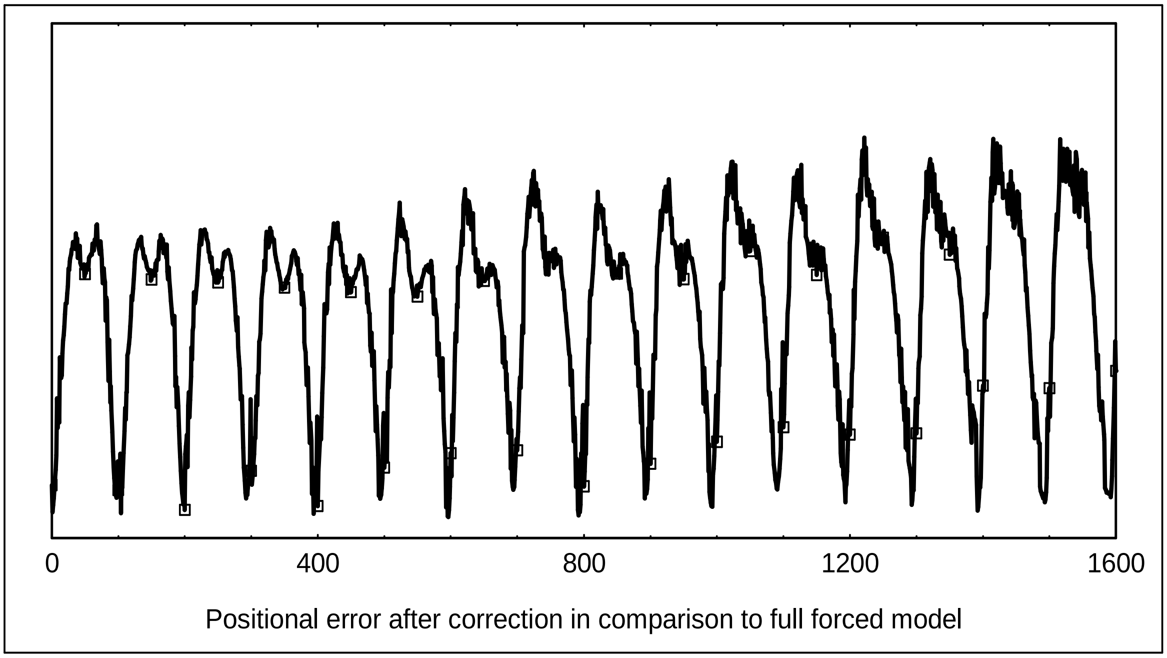

Figure 3: Positional difference

Absence of any secular drift is noticed while the periodic motion is less than 8kms. The periodic variations need not be corrected. Selection of start time for deriving the correcting fit is the cause for the periodic nature not being identical. Also a match between the osculating and this corrected motion once in every orbit. The motion over 1500 minutes corresponds to one day operations.

The proposed orbital motion is seen to retain a PCO, using an initial condition in Figure 4. The projected distance along the yz plane in the LVLH axes is observed steady without periodic effects over a day. State Transition Matrix and initialization is pending.

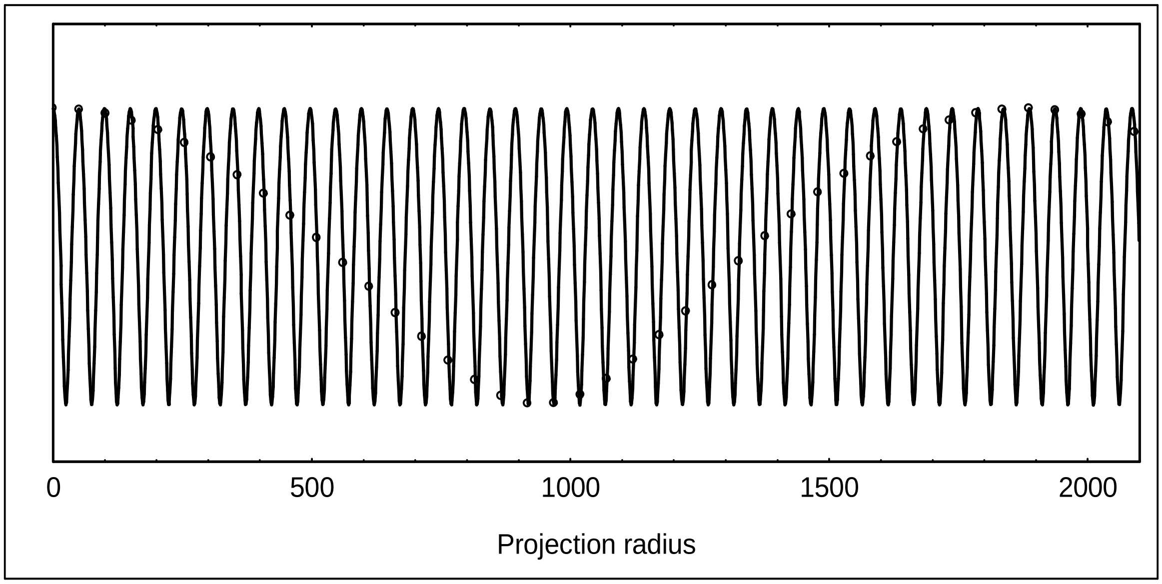

Figure 4

4. Conclusions

Relative motion by geometric approach for LEO satellites that accommodates all perturbations is provided. A kinematic orbit determination is used to obtain the OD faster and accurately by using a simple batch filter. To arrest the along-track secular variation in the full forced motion with respect to the oblate earth motion, the argument of latitude is appropriately corrected. This correction is extended to the subsequent orbits. Relative motion significantly captures the secular effects of all perturbations. At the start of each orbit the OD and guidance enables to estimate the extent of impulse maneuver required. It is shown that this can facilitate a direct use of the measurement. Simulation shows no unbounded variations. This approach can be followed in deriving the state transition matrix, which is an ongoing activity.

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