DISTRIBUTED ELECTROMAGNETIC CONTROL FOR SATELLITE SWARMS WITH LOCAL SENSING AND COMMUNICATING

HOU Zhendong[(1)]**, ZHANG Jinxiu**[(2)]**, and CAO Xibin**[(2)]

[(1)] *Harbin Institute of Technology, Room 507, B3 Block, Science Park, No.2 Yi Kuang Street, Nan Gang District, Harbin 150001, Heilongjiang Province, China P.R, hzd1988120@126.com*

[(2)] *Harbin Institute of Technology, Room 407, B3 Block, Science Park, No.2 Yi Kuang Street, Nan Gang District, Harbin 150001, Heilongjiang Province, China P.R, Jinxiu@hit.edu.cn*

Abstract: *The recently proposed system of satellite swarms is a novel development of traditional distributed space systems, which is supposed to exploit swarm intelligence and behaviors in nature to outperform other space systems for particular applications. However, due to the considerable number of satellites in a swarm, an effective maintaining method consuming as little fuel as possible is a challenge. This paper explores the application of electromagnetic force in satellite swarms. Firstly, a type of satellite swarm consisting of multiple groups is presented. To sustain the swarm, the inter-group control is occasionally actuated based on thrusters, while the inner-group control is accomplished via electromagnetic force which can be generated without onboard fuels. Then, a control method applying artificial potential functions is proposed for the inter-group maintaining, which is made up of velocity planning and tracking, and based on the information obtained using local sensing and communicating. Moreover, a control scheme based on this method is proposed for the inter-group maintaining. Finally, the magnetic dipoles to implement the control method are derived via nonlinear optimization methods. Simulation results illustrate that using the proposed control method, the desired inner-group shape can be formed and maintained successfully, and the required magnetic dipoles can be acquired effectively by nonlinear optimization.*

Keywords: *Satellite Swarms, Electromagnetic Force, Artificial Potential Functions, Nonlinear Optimization.*

1. Introduction

The concept of satellite swarms has been appearing in literature recently [1-4]. However, as most scientific innovations encountered, there haven’t existed a specific and acknowledged definition as well as its functionality. On the one hand, a satellite swarm can be simply regarded as a multi-satellite system without definite explanations about its functionality, composition and operation mode, and used as an alternative to satellite cluster or satellite formation [2]. Even so, it is generally employed in the situation that the number of satellites is substantial or the control requirements are relaxed [3, 4]. On the other hand, a satellite swarm can be defined as an exceptional distributed space system, which is inspired by various biological swarms in nature, e.g. bird flocking, animal herding, fish school, ant colonies [1]. In [1], the concept is explored from several typical properties of a natural swarm (e.g. robustness, redundancy, large area coverage, the lack of a hierarchical command structure, limited processing power per unit and selforganization). Moreover, the systems engineering problem taking into account its distinctiveness is analyzed in [3]. Despite of its immaturity, several projects of satellite swarms have been proposed to accomplish the missions for which other systems can't be competent, such as the OLFAR (Orbiting Low Frequency Array), a project aimed to develop a detailed system concept for space based very low frequency large aperture radio interferometric array [5], and the ANTS (Autonomous Nano-Technology Swarm) mission making use of satellite swarms to explore the asteroid belt [6].

Although satellite swarms would be promising in space application, there remain some challenges to be tackled, such as the effective control method to sustain the swarm consisting of considerable satellites. Due to the scale of swarm, the potential control method should only depend on information obtained by local sensing and communicating, and focus on minimizing the consumed fuel. For the former concern, artificial potential functions have been applied comprehensively, which imitate the swarm behaviors in nature [2, 4, 11]. And for the latter problem, the device of superconducting coils mounted on each satellite to generate electromagnetic force could be an excellent choice. This actuator can make the lifetime of system independent of the available onboard fuel, and exploit the theoretically inexhaustible solar energy instead [7]. Besides, the problem of plume contamination and thermal emission can be avoided as well [8].

However, the control applying electromagnetic force would make the dynamics of each satellite strongly coupled and require the solving of magnetic dipoles [9]. The difficulty would intensify as the number of satellites increases, which would be apparent in satellite swarms. Therefore, this paper firstly presents the analytical model of electromagnetic force and discusses its interacting traits. Based on the analysis, a type of satellite swarm suitable for the implementation of electromagnetic force control is proposed. The swarm is constructed by referring to the traits of swarm, and can be divided into multiple groups. Electromagnetic force is employed to control the inner-group geometry, whereas the inter-group's distance is ruled by traditional thrusters to compensate for the imperfection of electromagnetic force control. Then the relative orbital dynamics for this swarm is established based on the Hill's equations. Then a control method consisting of velocity planning and tracking is presented for the inter-group maintaining. The part of velocity planning is accomplished using artificial potential functions, and the part of velocity tracking is fulfilled by a feedback control algorithm. Furthermore, based on this method a control scheme is proposed for the inter-group maintaining. Finally, the magnetic dipoles to actuate the superconducting coils are calculated via nonlinear optimization methods.

2. Satellite Swarm Exploiting Electromagnetic Force

2.1 Electromagnetic Force Model

In this paper, the far-field model is adopted to formulate the electromagnetic force between satellites [9]. As each satellite with superconducting coils is approximated as a magnetic dipole in this model, the precision would degrade as the distance between satellites reduces and is applicable only when the ratio of inter-satellite distance to radius of superconducting coil exceeds 6~8 [10]. Even so, due to its simplicity the far-field model is applied extensively, especially in the field of control.

Using this model, the electromagnetic force exerted on satellite *i* by satellite *j* can be expressed as follows:

$$\boldsymbol{F}_{ij}^{m} = \frac{3\mu_m}{4\pi}\left(\frac{\boldsymbol{\mu}_i \bullet \boldsymbol{\mu}_j}{R_{ij}^5}\boldsymbol{R}_{ij} + \frac{\boldsymbol{\mu}_i \bullet \boldsymbol{R}_{ij}}{R_{ij}^5}\boldsymbol{\mu}_j + \frac{\boldsymbol{\mu}_j \bullet \boldsymbol{R}_{ij}}{R_{ij}^5}\boldsymbol{\mu}_i - 5\frac{\left(\boldsymbol{\mu}_i \bullet \boldsymbol{R}_{ij}\right)\bullet\left(\boldsymbol{\mu}_j \bullet \boldsymbol{R}_{ij}\right)}{R_{ij}^7}\boldsymbol{R}_{ij} \right) \tag{1}$$

where $\boldsymbol{\mu}_i, \boldsymbol{\mu}_j$ are the magnetic dipoles on satellite *i* and satellite *j* respectively. $\mu_m = 8\times10^{22}\,\mathrm{A}\bullet\mathrm{m}^2$ is Earth's dipole strength. $\boldsymbol{R}_{ij}$ represents the position vector from satellite *j* to satellite *i* and R_{ij} is its 2-norm.

Therefore, for a system consisting of *q* satellites with superconducting coils, the total electromagnetic force exerted on satellite *i* can be calculated by

$$\boldsymbol{F}_{i}^{m} = \sum_{j=1,\, j\neq i}^{q} \boldsymbol{F}_{ij}^{m} \tag{2}$$

For the design of suitable satellite swarm, two significant traits of electromagnetic force need to be analyzed. The first one is about the effective range of electromagnetic force. As shown in Eq. (1) , $\boldsymbol{F}_{ij}^{m}$ is proportional to R_{ij}^{-4} , which makes the electromagnetic interaction of two satellites flying with large distance trivial enough to be neglected. For instance, if R_{ij} increases to three times of the original, the force would reduce to 1/81 of the original. The second one is that the electromagnetic force is a type of interacting field force, which induces the resultant of force exerted on the swarm is null. Thus, the control fully dependent on electromagnetic force is invalid for maneuvering of overall swarm which may be essential in application.

2.2. Satellite Swarm Design

In this part, we conceive a type of satellite swarm applying electromagnetic force, which is based on the discussion in Section 2.1 and several traits of satellite swarm which is mentioned in the following description.

The swarm is supposed to be divided into multiple groups, and every group contains multiple satellites. The number of group can be determined according to the requirement of mission and may be as many as tens or hundreds. In contrast, the number of satellites in one group shouldn't exceed ten. Moreover, the distances between satellites belonged to one group are set to be at least three times less than that belonged to two distinct groups. For this swarm, the redundancy is represented by multitude groups, namely, some or all the groups can be designed identical to enhance the system's robustness as none of the groups is essential for the functionality [1]. The constraints on the scale of each group and the distinction of distances are contributed to reduce the degree of dynamics coupling to facilitate the application of electromagnetic force. In other words, only the satellites with superconducting coils in the same group are taken into account when evaluating the electromagnetic force exerted on any satellite.

In terms of maintaining the swarm, two types of control are required. The first one is called as inner-group control, and it is aimed to sustain the geometry formed by all the satellites in an identical group, e.g. a triangle formed by three satellites or a tetrahedron formed by four satellites. As generally hold for satellite swarm, the requirement for the precision of geometry sustaining can be relaxed [2]. The second one is called as inter-group control, and its major function is to confine the two adjacent groups within a predefined maximum range as well as to guarantee the distinction of distances presented above. It can be deduced that the inter-group control may be inessential for a relative short duration provided that the initial states of satellites are satisfying.

To implement the two control task, two kinds of actuators are employed accordingly. The inner-group control is dependent on the superconducting coils amounted on each satellite in the swarm to generate electromagnetic force to control the satellite’s relative motion. The inter-group control is propulsion based, but only one satellite among each group is equipped with thrusters for it. Therefore, the inner-group control would benefit from the fuel dependent trait of electromagnetic force especially when the control is required to be continuous, which is the situation of this paper. And the inter-group control wouldn’t consume too much fuel as it may be triggered occasionally. The major reason to use thrusters instead of fully electromagnetic actuating is that the latter can’t be capable of the control of the swarm’s entire motion.

The ability of local sensing and communicating is a significant trait of swarm. For this satellite swarm, each satellite has the identical sensing and communicating ability, moreover, the sensing ability and the communicating ability are indiscriminate, which means they can cover the same area. To meet the control requirement presented above, the sensing and communicating range should cover the maximum range defined for inter-group control. In fact, the satellites without thrusters unnecessarily sense or communicate in this far distance.

The following research on dynamics and control is based on this satellite swarm. For the convenience of description, the satellites responsible for inter-group control are denoted as guide satellites, and the remaining satellites are called as member satellites. l is denoted as the number of groups in a swarm, and q is denoted as the number of satellites belonged to an identical group. The guide satellite of group $k\left(k=1\sim l\right)$ is designated as S_1^k, and the member satellites can be designated as $S_j^k, j=2\sim q$. Figure 1 illustrates the structure of the proposed satellite swarm.

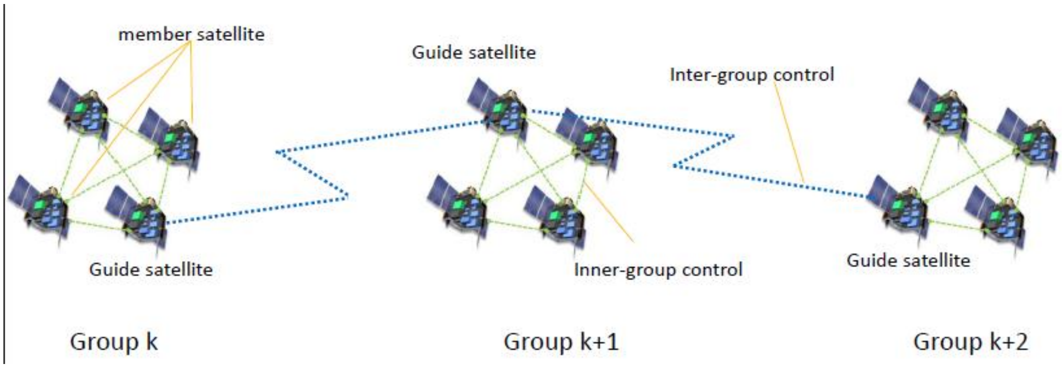

Figure 1. The Proposed Satellite Swarm

3 Relative Orbital Dynamics

The orbital dynamics of each satellite is established based on the classic Hill's equations which are effective when the distance from the satellite to its reference satellite is small enough and the reference satellite moves along an ideal circular orbit. As a satellite swarm may consists of tens or hundreds of satellites, its space distribution would be rather broad, which makes the accuracy of the Hill's equations degraded. Hence, this paper employs varied reference satellites for distinct groups of the swarm, and a virtual satellite located near the center of group would be an excellent choice.

The reference frame in which the orbital dynamics is expressed is defined as follows: the origin is located at the mass center of the reference satellite; the x axis is along the vector that points from the center of Earth to the reference satellite; the z axis points towards the orbital plane normal; the y axis completes the right hand system. Therefore, the orbital dynamics of S_i^k can be written as:

$$\ddot{\boldsymbol{x}}_i = \boldsymbol{A}_1 \boldsymbol{x}_i + \boldsymbol{A}_2 \dot{\boldsymbol{x}}_i + \boldsymbol{u}_i + \boldsymbol{d}_i \quad i = 1 \sim q$$
$$\boldsymbol{A}_1 = \begin{bmatrix} 3n^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -n^2 \end{bmatrix}, \boldsymbol{A}_2 = \begin{bmatrix} 0 & 2n & 0 \\ -2n & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3}$$

where n is the angular velocity of the reference satellite defined for group *k*, $\boldsymbol{x}_i$ represents the position vector of S_i^k, $\boldsymbol{u}_i$ is the control acceleration exerted on S_i^k, and $\boldsymbol{d}_i$ represents the disturbance acceleration, including the influence by various types of perturbations and the model error of the Hill's equations.

However, due to the interaction of electromagnetic force, the orbital dynamics of all the satellites in group *k* are coupling [7]. Thus, the control design in the following section would violate the coupling relationship if Eq. (3) is used directly.

To avoid this problem, the orbital dynamics of each member satellite is subtracted by that of their guide satellite mathematically, which derives the following model:

$$\ddot{\boldsymbol{x}}_{i1} = \boldsymbol{A}_1 \boldsymbol{x}_{i1} + \boldsymbol{A}_2 \dot{\boldsymbol{x}}_{i1} + \boldsymbol{u}_{i1} + \boldsymbol{d}_{i1} \quad i = 2 \sim q \tag{4}$$

where $\boldsymbol{x}_{i1}=\boldsymbol{x}_i-\boldsymbol{x}_1, \boldsymbol{u}_{i1}=\boldsymbol{u}_i-\boldsymbol{u}_1, \boldsymbol{d}_{i1}=\boldsymbol{d}_i-\boldsymbol{d}_1$. Therefore, instead of deriving the control acceleration of each satellite belonged to the identical group directly, $\boldsymbol{u}_{i1}$ is obtained using the control method in Section 4. Then by combining the coupling equation introduced in Section 5, the reasonable $\boldsymbol{u}_i$ for each satellite can be determined.

4. Control Strategy Based on Artificial Potential Functions

In this section, a control method is proposed for the inner-group geometry forming and maintaining. The method can be divided into two parts: velocity planning and tracking. The former can generate the desired velocity of each satellite according to its actual position with respect to other satellites in the same group. The latter gives the required acceleration to track the planned velocity (The tracking algorithm also requires the actual velocity of each satellite with respect to other satellites in the same group). In addition, an inter-group control scheme is proposed via an extension of the above method.

4.1 Velocity Planning

The process of velocity planning is accomplished by using one type of artificial potential functions which is enlightened by the intuitive behaviors of natural creatures in a swarm. Specifically, for two satellites, the function simulates the behavior of approaching when their distance is large and the behavior of separating when the distance is small. The function has been widely used to achieve swarm aggregation [1-2], which can be written as follows:

$$\boldsymbol{g}(\boldsymbol{y})=-\boldsymbol{y}\left[a-b\exp\left(-\frac{\boldsymbol{y}^T\boldsymbol{y}}{c}\right)\right] \tag{5}$$

where a,b,c are positive real numbers, and $\boldsymbol{y}$ is a vector variable to determine behaviors. In terms of satellite swarms, $\boldsymbol{y}$ can be the relative position with respect to other satellites or particular flags of surroundings.
It can be seen that if one satellite is actuated by the velocity defined as Eq. (5), the satellite would arrive at the equilibrium of $\boldsymbol{g}(\boldsymbol{y})=\boldsymbol{0}$ which can be calculated by the following equation:

$$d_{eq}=\sqrt{\ln\frac{b^c}{a}} \tag{6}$$

As discussed in Section 2, Eq. (4) is applied for electromagnetic force control. Thus, for a group with q satellites, only $q-1$ relative velocities can be planned. Using the artificial potential function presented by Eq. (5), the desired velocity field assigned to group *k* is formulated by

$$\boldsymbol{v}_{i1} = \sum_{j=2,\, j\neq i}^{q} \boldsymbol{g}_{ij}\left(\boldsymbol{x}_{ij}\right) + \boldsymbol{g}_i\left(\boldsymbol{x}_{i1}\right) \quad i = 2 \sim q \tag{7}$$

where

$$\begin{gathered} \boldsymbol{g}_{ij}\left(\boldsymbol{x}_{ij}\right) = -\boldsymbol{x}_{ij}\left[a_{ij} - b_{ij}\exp\left(-\frac{\boldsymbol{x}_{ij}^T\boldsymbol{x}_{ij}}{c_{ij}}\right)\right] \\ \boldsymbol{g}_i\left(\boldsymbol{x}_{i1}\right) = -\boldsymbol{x}_{i1}\left[a_i - b_i\exp\left(-\frac{\boldsymbol{x}_{i1}^T\boldsymbol{x}_{i1}}{c_i}\right)\right] \\ \boldsymbol{x}_{ij} = \boldsymbol{x}_{i1} - \boldsymbol{x}_{j1} \end{gathered} \tag{8}$$

According to the above discussion on $\boldsymbol{g}(\boldsymbol{y})$, it is evident that $\boldsymbol{g}_{ij}\left(\boldsymbol{x}_{ij}\right)$ is used to drive and keep the distance between $S_i^{O_k}$ and $S_j^{O_k}$ to a design value determined by a_{ij}, b_{ij} and c_{ij}, and $\boldsymbol{g}_i\left(\boldsymbol{x}_i\right)$ is to maintain the distance between $S_i^{O_k}$ and O_k determined by a_i, b_i and c_i. Therefore, the set of $\boldsymbol{v}_{i1}$ can instruct all the satellites inside group *k* to behave concurrently to arrive at an equilibrium of the group. And the inner-group control objective of geometry forming and keeping can be achieved by planning an appropriate equilibrium in advance.

Although the velocity field presented by Eq. (7) is biological swarm inspired, and has distinct physical explanation, some essential mathematical analysis should be complemented to confirm the reliability and applicability of the method. To this end, the following Lyapunov function is constructed:

$$J\left(\boldsymbol{x}\right) = \sum_{i=2}^{q-1}\sum_{j>i}^{q} J_{ij} + \sum_{i=2}^{q} J_i \tag{9}$$

where

$$\begin{gathered} J_{ij} = a_{ij}\left\|\boldsymbol{x}_{ij}\right\|^2 + b_{ij}c_{ij}\exp\left(-\frac{\left\|\boldsymbol{x}_{ij}\right\|^2}{c_{ij}}\right) \\ J_i = a_i\left\|\boldsymbol{x}_{i1}\right\|^2 + b_i c_i\exp\left(-\frac{\left\|\boldsymbol{x}_{i1}\right\|^2}{c_i}\right) \\ \boldsymbol{x} = \left[\boldsymbol{x}_{21}^T, \cdots, \boldsymbol{x}_{q1}^T\right]^T \end{gathered} \tag{10}$$

Therefore, the gradient of $J\left(\boldsymbol{x}\right)$ with respect to each $\boldsymbol{x}_{k1}, k = 2 \sim q$ is given as follows:

$$\begin{aligned}\nabla_{\boldsymbol{x}_{k1}} J\left(\boldsymbol{x}\right) &= \sum_{j>k}^{q} \nabla_{\boldsymbol{x}_{k1}} J_{kj} + \sum_{i=2}^{k-1} \nabla_{\boldsymbol{x}_{k1}} J_{ik} + \nabla_{\boldsymbol{x}_{k1}} J_{k} \\ &= -2\sum_{j>k}^{q} \boldsymbol{g}_{kj}\left(\boldsymbol{x}_{kj}\right) + 2\sum_{i=2}^{k-1} \boldsymbol{g}_{ik}\left(\boldsymbol{x}_{ik}\right) - 2\boldsymbol{g}_{k}\left(\boldsymbol{x}_{k1}\right)\end{aligned} \tag{11}$$

It is apparent that $\boldsymbol{g}_{ij}\left(\boldsymbol{x}_{ij}\right)$ and $\boldsymbol{g}_{ji}\left(\boldsymbol{x}_{ji}\right)$ are used for the same purpose. Thus, one intuitive choice of their parameters complies with the following equations:

$$a_{ij} = a_{ji} \quad b_{ij} = b_{ji} \quad c_{ij} = c_{ji} \tag{12}$$

For this choice, we can derive the following result from Eq. (11):

$$\begin{aligned}\nabla_{\boldsymbol{x}_{k1}} J\left(\boldsymbol{x}\right) &= -2\sum_{j>k}^{q} \boldsymbol{g}_{kj}\left(\boldsymbol{x}_{kj}\right) - 2\sum_{i=2}^{k-1} \boldsymbol{g}_{ki}\left(\boldsymbol{x}_{ki}\right) - 2\boldsymbol{g}_{k}\left(\boldsymbol{x}_{k1}\right) \\ &= -2\boldsymbol{v}_{k1}\end{aligned} \tag{13}$$

Hence, the time derivative of the Lyapunov function along the desired velocity field can be expressed as:

$$\dot{J}\left(\boldsymbol{x}\right) = \sum_{k=2}^{q} \left[\nabla_{\boldsymbol{x}_{k1}} J\left(\boldsymbol{x}\right)\right]^{T} \dot{\boldsymbol{x}}_{k1} = -2\sum_{k=2}^{q} \left\|\dot{\boldsymbol{x}}_{k1}\right\|^{2} \leq 0 \tag{14}$$

Then, Using the LaSalle's Invariance Principle, we can deduce that as $t \to \infty$ the variable $\boldsymbol{x}$ converges to the largest invariant subset of the set defined as:

$$\Omega = \left\{\boldsymbol{x} \mid \dot{J}\left(\boldsymbol{x}\right) = 0\right\} = \left\{\boldsymbol{x} \mid \dot{\boldsymbol{x}} = 0\right\} \tag{15}$$

However, we can't guarantee that the largest invariant subset of Ω only comprises the equilibrium defined by Eq. (6). There may exist several local minima that make the ultimately formed geometry isn't unique [12]. Although the problem can be resolved by selecting proper parameters, the sophisticated methods to determine them one time have not been presented as far as we know. One alternative to escape a local minimum is to change the original parameters when the satellite evaluates its desired velocity to be zero [13].

In this paper, we attempt to reduce the probability of occurrence of falling into local minima via exploiting the diversity and randomicity of parameters. As any local minimum only appears when all the satellites' positions and parameters satisfy certain equations ($\boldsymbol{v}_{k1} = \boldsymbol{0}, k = 2 \sim q$), we can anticipate that the problem is almost impossible to be encountered during the process of geometry forming for random initial positions and random parameters. Specifically, the method comprises two steps: 1) According to the desired shape, each member satellite decides its desired distances with respect to other satellites and generates its own parameters randomly using the onboard computer program; 2) any two member satellites communicate with each other to negotiate a consistent result on their related parameters (e.g. a_{ij} and a_{ji}) to meet Eq. (12).

4.2 Velocity Tracking

In order to track the velocities defined by Eq. (7), the following feedback control algorithm is applied [2]:

$$\boldsymbol{u}_{i1} = \boldsymbol{K}_i \left(\boldsymbol{v}_{i1} - \dot{\boldsymbol{x}}_{i1} \right) + \boldsymbol{a}_{i1} - \left(\boldsymbol{A}_1 \boldsymbol{x}_{i1} + \boldsymbol{A}_2 \dot{\boldsymbol{x}}_{i1} \right) \quad i = 2 \sim q \tag{16}$$

where $\boldsymbol{K}_i$ is a constant vector, $\boldsymbol{x}_{i1}$ and $\dot{\boldsymbol{x}}_{i1}$ are the measured positions and velocities respectively, $\boldsymbol{v}_{i1}$ and $\boldsymbol{a}_{i1}$ are their planned counterparts. $\boldsymbol{a}_{i1}$ can be derived by differentiating $\boldsymbol{v}_{i1}$, which is expressed as

$$\boldsymbol{a}_{i1} = \sum_{j=2, j \neq i}^{q} \tilde{\boldsymbol{g}}_{ij} \left(\boldsymbol{x}_{ij} \right) + \tilde{\boldsymbol{g}}_i \left(\boldsymbol{x}_{i1} \right) \tag{17}$$

where

$$\begin{aligned} \tilde{\boldsymbol{g}}_{ij} \left(\boldsymbol{x}_{ij} \right) &= -\dot{\boldsymbol{x}}_{ij} \left[a_{ij} - b_{ij} \exp\left(-\frac{\boldsymbol{x}_{ij}^T \boldsymbol{x}_{ij}}{c_{ij}} \right) \right] - \boldsymbol{x}_{ij} \left[\frac{2b_{ij}}{c_{ij}} \exp\left(-\frac{\boldsymbol{x}_{ij}^T \boldsymbol{x}_{ij}}{c_{ij}} \right) \dot{\boldsymbol{x}}_{ij}^T \boldsymbol{x}_{ij} \right] \\ \tilde{\boldsymbol{g}}_{i1} \left(\boldsymbol{x}_{i1} \right) &= -\dot{\boldsymbol{x}}_{i1} \left[a_i - b_i \exp\left(-\frac{\boldsymbol{x}_{i1}^T \boldsymbol{x}_{i1}}{c_i} \right) \right] - \boldsymbol{x}_{i1} \left[\frac{2b_i}{c_i} \exp\left(-\frac{\boldsymbol{x}_{i1}^T \boldsymbol{x}_{i1}}{c_i} \right) \dot{\boldsymbol{x}}_{i1}^T \boldsymbol{x}_{i1} \right] \end{aligned} \tag{18}$$

4.3 Inter-group control scheme

Although the control method in this section only illustrates its applicability for inner-group geometry forming and maintaining, the inter-group control can inherit it conveniently. Therefore, we propose the following control scheme.

To apply this scheme, two trigger ranges are defined for each guide satellite. The smaller one denoted as $RA_{\min}$ should exceed the satellite's inter-group control minimal allowed range denoted as RA_d. The larger one denoted as RA_{max} should be confined inside the satellite's inter-group control maximal allowed range denoted as RA_{sc}. The thrusters on one guide satellite fire only when other guide satellites intrude into RA_{min} or escape RA_{max}. And the maneuvering acceleration is determined according to a procedure of velocity planning and tracking alike that for inner-group control. The desired distance d_{eq} is supposed to be a value close to the middle of RA_{min} and RA_{max}. Figure 2 demonstrates the above control scheme for inter-group maintaining.

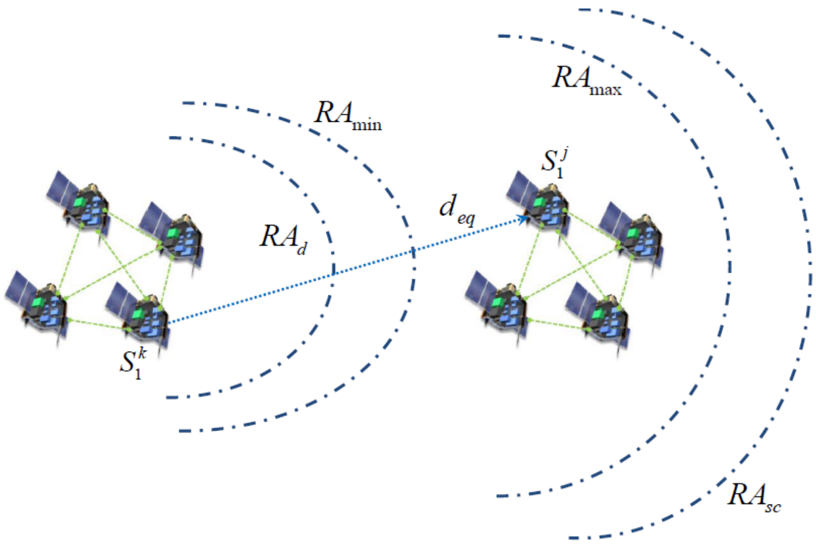

Figure 2. The Inter-group Control Scheme

It is manifest that the inter-group control is actuated occasionally. Whereas, to minimize the fuel consumption as much as possible, further study should be carried out for the design of guide satellites' initial positions, the definition of trigger ranges and the selecting of parameters in artificial potential functions.

5. Derivation of Magnetic Dipoles

The control method for inner-group in Section 4 can only give the desired acceleration. In terms of electromagnetic force control, we need to obtain the appropriate magnetic dipoles to actuate the superconducting coils to generate the desired acceleration.
Assuming that the mass of each satellite is identical and denoted by *m*, the following equations hold:

$$\begin{aligned} &\boldsymbol{F}_i - \boldsymbol{F}_1 = m\boldsymbol{u}_{i1} \quad i = 2,\cdots,q \\ &\sum_{i=1}^{q} \boldsymbol{F}_i = \boldsymbol{0} \end{aligned} \tag{19}$$

where $\boldsymbol{F}_i$ is the electromagnetic force exerted on satellite S_i^k.

Equation (19) is equivalent to the following expression:

$$
\left.\begin{aligned}
\boldsymbol{F}_1 &= -\frac{m}{q}\sum_{i=2}^{q}\boldsymbol{u}_{i1} \\
\boldsymbol{F}_i &= \frac{m}{q}\left(q\boldsymbol{u}_{i1}-\sum_{j=2}^{q}\boldsymbol{u}_{j1}\right)
\end{aligned}\right. \quad i=2,\cdots,q \tag{20}
$$

Although there are $3q$ scalar equations in Eq. (20), we can only use $3q-3$ equations from them to derive the magnetic dipoles as they are dependent. In this paper, the following equations are chosen as constraints of magnetic dipoles solving:

$$
\boldsymbol{F}_i\left(\boldsymbol{\mu}_1,\boldsymbol{\mu}_2,\cdots,\boldsymbol{\mu}_q\right)=\frac{m}{q}\left(q\boldsymbol{u}_{i1}-\sum_{j=2}^{q}\boldsymbol{u}_{j1}\right) \quad i=2,\cdots,q \tag{21}
$$

It is evident that Eq. (21) has $3q$ unknown variables, which makes the solution satisfying the acceleration requirement not unique. Thus, we attempt to exploit the diversity of solution to derive an optimal solution for a predefined cost function which is defined as follows:

$$
J\left(t_k\right)=\frac{1}{2}\sum_{i=1}^{q}\left\{\boldsymbol{\mu}_i^T\left(t_k\right)\boldsymbol{W}_{mi}\boldsymbol{\mu}_i\left(t_k\right)+\left[\boldsymbol{\mu}_i\left(t_k\right)-\boldsymbol{\mu}_i\left(t_{k-1}\right)\right]^T\boldsymbol{W}_{di}\left[\boldsymbol{\mu}_i\left(t_k\right)-\boldsymbol{\mu}_i\left(t_{k-1}\right)\right]\right\} \tag{22}
$$

where t_k and t_{k-1} represent two successive solving time, $\boldsymbol{W}_{mi}$ and $\boldsymbol{W}_{di}$ are both diagonal weighting matrices. The first term in Eq. (22) is to minimize the magnitude of all the magnetic dipoles and average the magnitude of each one, and the second term is to avoid unnecessary switching of the dipoles.

In this paper, we use the optimization toolkit in Matlab to solve the nonlinear optimization problem defined by Eq. (21) and Eq. (22). Although it may seem to require much computing, the solution should be continuous and once the solution in t_{k-1} is found, the solution in t_k is very close to this solution. Hence, setting the initial guess at each time step with the previously computed solution would result in a very rapid convergence to the desired solution.

6. Numerical Results

In this section the performance of the proposed inner-group control method and magnetic dipoles solving algorithm is examined by the geometry forming and maintaining for a specific satellite swarm described as Section 2.

The satellite swarm is made up of three groups, and each group consists of four satellites which are expected to form and maintain the geometry of regular tetrahedron with side length 15 m. The initial deployment of the swarm makes it possible to select three virtual reference satellites flying along an identical circular orbit with various true anomalies. The orbit altitude is 500 km, and the distance of two adjacent reference satellites is 1 km. The initial position of each satellite in its respective reference frame is given in Tab. 1.

Table 1. Initial Position of Each Satellite

	Satellite 1's position (m)	Satellite 2's position (m)	Satellite 3's position (m)	Satellite 4's position (m)
Group 1	-10, 0, 0	10, 10, 0	-5, 15, 0	0, -5, 15
Group 2	10, 0, 3	10, -10, -1	5, 15, 0	0, 5, -10
Group 3	5, 0, 0	15, -10, 0	8, -7, 0	0, 5, -10

The parameters of each artificial potential function for the swarm's velocity planning are identical without violation of Eq. (6), namely, $a_{ij} = a_i = 0.01 \times e^{-1}$, $b_{ij} = b_i = 0.01$, $c_{ij} = c_i = 15^2$. Meanwhile, $\boldsymbol{K}_i = \mathrm{diag}(0.01, 0.01, 0.01)$ is chosen for the velocity tracking of each member satellite.

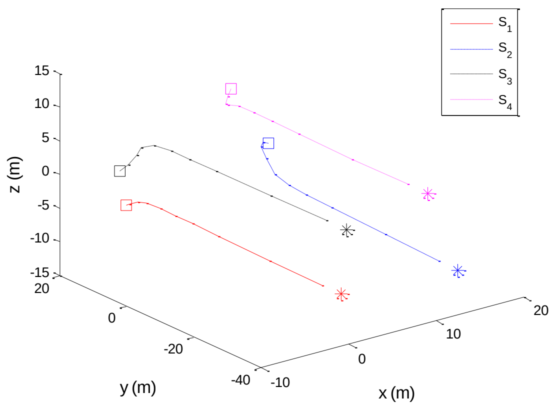

Figure 3. Trajectories of Group 1 for Geometry Forming and Maintaining

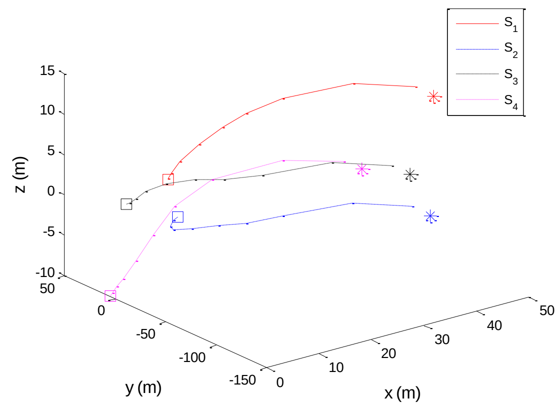

Figure 4. Trajectories of Group 2 for Geometry Forming and Maintaining

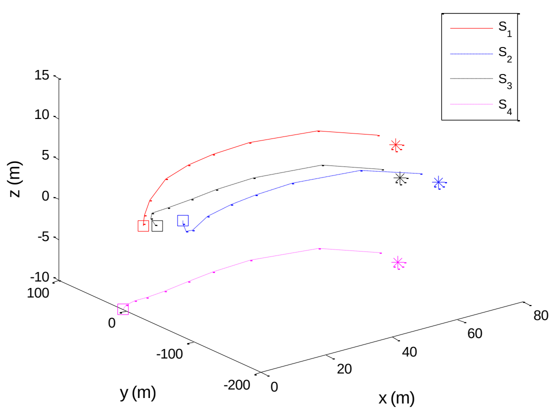

Figure 5. Trajectories of Group 3 for

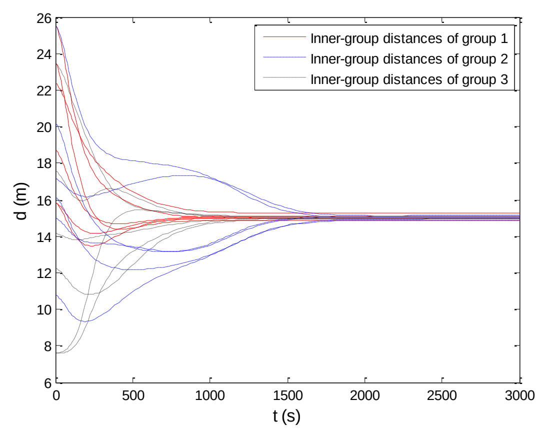

Figure 6. Varying of Inter-satellite

Geometry Forming and Maintaining **Distances of Each Group**

To solve the magnetic dipoles, the mass of each satellite is identical with the value of 300 kg. Every diagonal element of $\boldsymbol{W}_{mi}$ is set to be 1, and that is 50 for $\boldsymbol{W}_{di}$.

The trajectories of geometry forming and maintaining for each group are depicted in Figs. 3~5 with the initial positions marked by squares and the terminal positions marked by asterisks, and the varying of inter-satellite distances of each group is illustrated in Fig. 6. It can be seen that by using the proposed control method consisting of velocity planning and velocity tracking, the satellites in each group can be actuated to form the desired geometry of regular tetrahedron from the initial positions respectively, and the maximum deviation during the phase of geometry maintaining is below 0.3 m.

While the inter-group control based on thrusters are not actuated due to the relaxed initial positions for simulation, Table. 2 gives the initial and terminal centers of mass of each group to show some phenomenon. It can be seen that although the electromagnetic force can't shift the center of mass of any group, none of the three groups can preserve their initial values due to Earth's gravitation. And for various initial deployments, the drifts are markedly diverse. Hence, further research should be focused on the initialization of each group to facilitate the natural maintaining of inter-group distances to avoid the unwelcome inter-group control as far as possible.

Table 2. Initial and Terminal Centers of Mass of Each Group

	Initial position (m)	Terminal position (m)
Group 1	-1.25, 5, 3.75	9.06, -29.742, -6.03
Group 2	6.25, 2.5, -2	36.20, -102.34, 6.21
Group 3	7, -3, -2.5	52.45, -162.72, 5.22

The required magnetic dipoles to implement the above inner-group control are shown in Fig. 7~9. It can be seen that the magnitudes of all the magnetic dipoles are restricted below $5\times10^{5}\,\mathrm{A}\bullet\mathrm{m}^{2}$, and the magnetic dipoles vary smoothly over time without significant switch of vector direction.

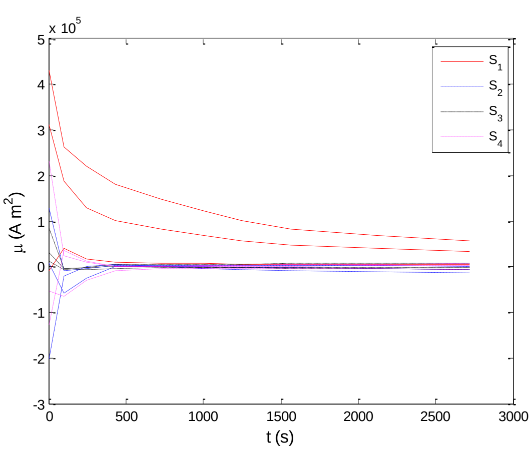

Figure 7. Time histories of the magnetic dipoles in Group 1

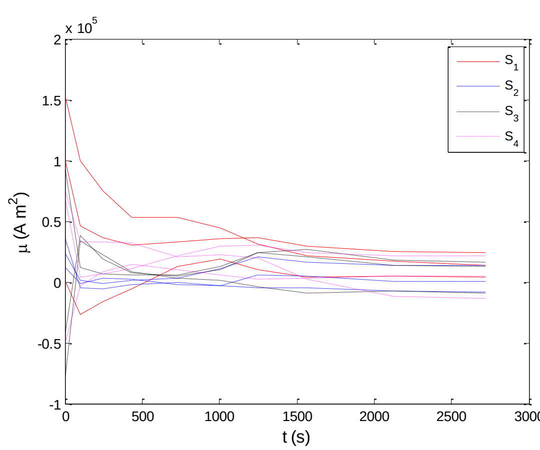

Figure 8. Time histories of the magnetic dipoles in Group 2

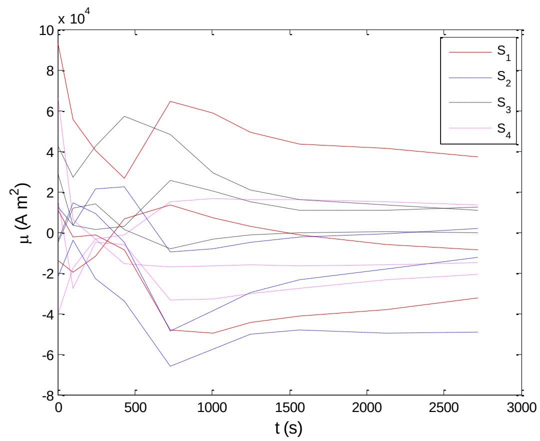

Figure 9. Time histories of the magnetic dipoles in Group 3

7. Conclusions

As one of the diverse solutions to the reduction of fuel consumption for the maintaining of satellite swarms, the electromagnetic force based control method is proposed. Firstly, a specific satellite swarm structure is designed as well as its control requirement to facilitate the control implementation applying electromagnetic force by decreasing the degree of dynamic coupling. Then a control strategy synthesizing the thrusters for inter-group control and the superconducting coils for inner-group control is presented. The inner-group control algorithm is composed of velocity planning, which is inspired by natural swarm behaviors and constructed by artificial potential functions, and velocity tracking consisting of Hill's equations based feedforward and feedback tracking. The inter-group control is designed to be actuated by two predefined trigger ranges and employ the algorithm similar to that for inner-group control. Finally, nonlinear optimization methods are used to derive the desired magnetic dipoles. The feasibility and effectiveness of the proposed inner-group control method and magnetic dipoles solving method are verified by simulation. Further research should be conducted on the determination of trigger ranges, the initial deployment and the parameter optimization of the artificial potential functions.

8. References

[1] Verhoeven, C.J.M., Bentum, M.J., Monna et al., "On the Origin of Satellite Swarms," Acta Astronautica, Vol. 68, No.7-8, 2011, pp. 1392-1395.

[2] Nag, S., Summerer, L., "Behaviour based, autonomous and distributed scatter manoeuvres for satellite swarms," Acta Astronautica, Vol. 82, No.1, 2013, pp. 95-109.

[3] Engelen, S., Gill, E.K.A., Verhoeven, C.J.M., "Systems Engineering Challenges for Satellite Swarms," Aerospace Conference, 2011 IEEE, pp. 1-8.

[4] Izzo, D., Pettazzi, L., "Autonomous and Distributed Motion Planning for Satellite Swarm," Journal of Guidance, Control, and Dynamics, Vol. 30, No.2, 2007, pp. 449-459.

[5] Rajan, R.T., Engelen, S., Bentum, M., Verhoeven, C., “Orbiting low frequency array for radio astronomy,” IEEE Aerospace Conference, March 2011, pp. 1–11.

[6] Truszkowski, W., Rash, J., Rouff, C., Hinchey, M., “Asteroid Exploration with Autonomic Systems,” Proceedings of the 11th IEEE International Conference and Workshop on the Engineering of Computer-Based Systems, May 2004, pp. 484–489.

[7] Ahsun, U., Miller, D.W., Ramirez, J.L., “Control of Electromagnetic Satellite Formations in Near-Earth Orbits,” Journal of Guidance, Control, and Dynamics, Vol. 33, No. 6, 2010, pp. 1883-1891.

[8] Kwon, D.W., “Electromagnetic Formation Flight of Satellite Arrays,” Massachusetts, U.S.: Massachusetts Institute of Technology, 2005.

[9] Schweighart, S.A., Sedwick, R.J., “Explicit Dipole Trajectory Solution for Electromagnetically Controlled Spacecraft Clusters,” Journal of Guidance, Control, and Dynamics, Vol. 33, No. 4, 2010, pp. 1225-1235.

[10] Schweighart, S.A., Sedwick, R.J., “Propellantless Formation Flight Operations using Electromagnetic Formation Flight,” SpaceOps 2006 Conference, Huntsville, Alabama, 2006, pp. 1-38.

[11] Gazi, V., “Swarm Aggregations Using Artificial Potentials and Sliding Mode Control,” IEEE Transactions on Robotics, Vol. 21, No.6, 2005, pp. 1208-1214.

[12] Gazi, V., Passim, K.M., “Stability Analysis of Social Foraging Swarms Combined Effects of Attractant repellent Profiles,” Proceedings of the 41st IEEE Conference on Decision and Control, Vol. 3, 2002, pp. 2848-2853.

[13] Izzo, D., Pettazzi, L., “Equilibrium Shaping: Distributed Motion Planning for Satellite Swarm,” Proceedings of the 8th International Symposium on Artificial Intelligence, Robotics and Automation in Space, SP-603, ESA Publications Division, Noordwijk, The Netherlands, Sept. 2005.