ROBUST ESTIMATION FOR RELATIVE POSITION OF SPACECRAFT FORMATION FLYING

Wang Wei[(1)], **Huang Wende**[(2)], **Xi Xiaoning**[(3)]
[(1)(2)]*College of Mechatronic Engineering and Automation, NUDT, 410073, Changsha, China, +8613508486238, wangwei4213440@163.com*
[(3)]*College of Aerospace and Material Engineering, NUDT, 410073, Changsha, China, +8613308492496, xilily0704@126.com*

Abstract: *One of the fundamental issues of spacecraft formation flying is the determination of the relative position between the satellite vehicles within the formation. The application of GNSS in spacecraft formation flying operates in less than ideal environment where there is a high probability of faults occurring. A three satellites formation flying scenario in near-earth with GPS-based navigation has been simulated. A robust estimator, so-called IGG3 scheme, has been implement to mitigate the influence of outlier observations in relative position of satellite formation flying based on GNSS. Computation results demonstrated that, the least-square estimator and the IGG3 robust estimator are equivalent when there are no outliers; the least-square estimator is influenced by outlier, but the IGG3 robust estimator is not heavily affected by outlier when 0.3-0.5 cycle of carrier phase outlier observation appears. The relative position accuracy of robust estimator is almost about 0.03m in both situations.*

Keywords: *Spacecraft Formation Flying, Relative Position, Robust Estimation*

1. Introduction

Spacecraft formation flying, considered as a key technology for advanced space missions, has received more attention. Satellite formation flying to near-Earth applications provide advanced science opportunities that cannot, or not easily, be realized with single spacecraft[1]. One of the fundamental issues of spacecraft formation flying is the determination of the relative position between the satellite vehicles within the formation, which is more important for certain formation flying missions, such as those proposed for the earth-observation space distributed radar whose accuracy of relative position requires centimeter level[2][3]. Most researches into determining both absolute and relative positions between vehicles have involved using GNSS (e.g. GPS)[4]. It is often considered the carrier phase measurements of GNSS as the primary instruments for precise relative navigation in satellite formation flying missions. The application of GNSS in spacecraft formation flying operates in less than ideal environment where there is a high probability of faults occurring, and it increases the rate at which a receive experiences one or more faulty observations.

The abnormal phase measurements (or outliers) produce unfavorable effects on the final estimate of the relative position. As we know, all of the estimators based on the least squares and the maximum likelihood in standard normal distribution are not robust. Any faults or outliers in the observations could dramatically affect the estimates of unknown parameters and the variance components. With the growing desire to obtain a highly reliable position of formation satellites, there is a mounting need to detect and mitigate the presence of faults or outliers. The approaches to controlling the outlier influence fall into two broad categories: outlier identification and robust estimation.

Our main aim in this paper is to investigate the application of robust estimation in the presence of abnormal phase measurements of GNSS when relative position estimation is executed. A three satellites formation flying scenario in near-earth with GPS-based navigation has been simulated. The least-square estimator and the so-called IGG3 robust estimator were carried out when there was none and one outlier within any observation in any epoch. The computation results demonstrated that the least-square estimator and the IGG3 robust estimator are equivalent when there are no outliers, the least-square estimator is influenced by outlier, while the IGG3 robust estimator is not heavily affected by outlier.

2. Measurement Models and Robust Estimator

2.1 Measurement Models

The measurement of carrier phase between formation vehicle i and GNSS satellite k is defined:

$$f_i^k = r_i^k + c \cdot dt_i - c \cdot dt^k - I_i^k + N_i^k + n_i^k \tag{1}$$

Where

r_i^k = geometric range between vehicle i and GNSS satellite k

dt_i = clock offset for vehicle i

dt^k = clock offset for GNSS satellite k

I_i^k = ionosphere delay

N_i^k = carrier integer ambiguity between i and k

n_i^k =other noises(including receiver-based uncertainties, multipath, and unmodeled environment)

The term r_i^k is $\left|\mathbf{X}^k(t^s) - \mathbf{X}_i(t_r)\right|$, where $\mathbf{X}^k(t^s)$ is the GNSS satellite position at the time the signal was sent and $\mathbf{X}_i(t_r)$ is the vehicle position at the time the signal was received.

The inter-vehicle single difference between vehicle i and j with respect to GNSS satellite k is:

$$\Delta f_{ij}^{k} = r_{j}^{k} - r_{i}^{k} + c \cdot dt_{j} - c \cdot dt_{i} + \Delta N_{ij}^{k} + \Delta n_{ij}^{k} \tag{2}$$

Where

$\Delta f_{ij}^{k} = f_{j}^{k} - f_{i}^{k}$, $\Delta N_{ij}^{k} = \Delta N_{j}^{k} - \Delta N_{i}^{k}$, $\Delta n_{ij}^{k} = \Delta n_{j}^{k} - \Delta n_{i}^{k}$.

The inter-vehicle double difference between vehicle i and j with respect to GNSS satellite k and l is:

$$\nabla\Delta f_{ij}^{kl} = (r_{j}^{l} - r_{i}^{l}) - (r_{j}^{k} - r_{i}^{k}) + \nabla\Delta N_{ij}^{kl} + \nabla\Delta n_{ij}^{kl} \tag{3}$$

Where

$\nabla\Delta f_{ij}^{k} = \Delta f_{ij}^{l} - \Delta f_{ij}^{k}$, $\nabla\Delta N_{ij}^{kl} = \Delta N_{ij}^{l} - \Delta N_{ij}^{k}$, $\nabla\Delta n_{ij}^{kl} = \Delta n_{ij}^{l} - \Delta n_{ij}^{k}$.

2.2 Robust Estimator

One of standard estimation algorithms that exist for solving the problem of estimating position is weighted least squares. It does not rely on a model of the system dynamics, but simply finds the position solution that yields the best fit for each new set of data. In general, the error equation which from Eq. 2 or Eq. 3 can be expressed as

$$\mathbf{V} = \mathbf{AX} - \mathbf{L} \tag{4}$$

Where

$\mathbf{A}$ = the design matrix
$\mathbf{X}$ = the estimated vector of unknown parameters (receiver position)
$\mathbf{L}$ = the so-called observation vector
$\mathbf{V}$ = the residual vector of $\mathbf{L}$.

The LS estimator of $\mathbf{X}$ is expressed as

$$\hat{\mathbf{X}} = (\mathbf{A}^{\mathrm{T}}\mathbf{PA})\mathbf{A}^{\mathrm{T}}\mathbf{PL} \tag{5}$$

and the estimator of the variance of unit weight is

$$\hat{s}^{2} = \frac{\mathbf{V}^{\mathrm{T}}\mathbf{PV}}{n-m} \tag{6}$$

where $\mathbf{P}$ is the observation weight matrix, n is the number of observations, and m is the number of unknown parameters.

The LS is not robust, and the LS estimates of $\mathbf{X}$ will be affected if some observations have apparent outliers. To reduce the influence of these outliers, the IGG3 scheme of robust estimator based on the principle of M estimation was derived[5]

$$\hat{\mathbf{X}} = (\mathbf{A}^{\mathrm{T}}\bar{\mathbf{P}}\mathbf{A})\mathbf{A}^{\mathrm{T}}\bar{\mathbf{P}}\mathbf{L} \tag{7}$$

where $\bar{\mathbf{P}}$ is so-called equivalent weight matrix with elements $\bar{p}_{ij}$. Obviously, the robust estimator based on the IGG3 scheme has the same formation as that based on least-square (LS) principle.

The $\bar{p}_{ij}$ is called equivalent weight, and it is expressed with IGG3 scheme as

$$\bar{p}_{ij} = \begin{cases} p_{ij} & |v'_j| \le k_0 \\ p_{ij} \dfrac{k_0}{|v'_j|} \left(\dfrac{k_1 - |v'_j|}{k_1 - k_0} \right)^2 & k_0 < |v'_j| \le k_1 \\ 0 & |v'_j| > k_1 \end{cases} \tag{8}$$

Where $v'_j = v_j / s_j$ is a standardized residual, k_0 and k_1 are two experienced constants, and they may be chosen on the objective requirements of the problem in theory or in practice. Usually, k_0 is chosen to be 1.0~1.5, and k_1 is chosen to be 2.5~3.0[5].

3. Computation and Comparison

To compare the performance against the outliers of the LS estimator and robust estimator, the mathematical simulation and has been conducted. A three satellites formation flying scenario in near-Earth with GPS-based navigation has been simulated. Figure 1 shows the number of GPS satellites visible.

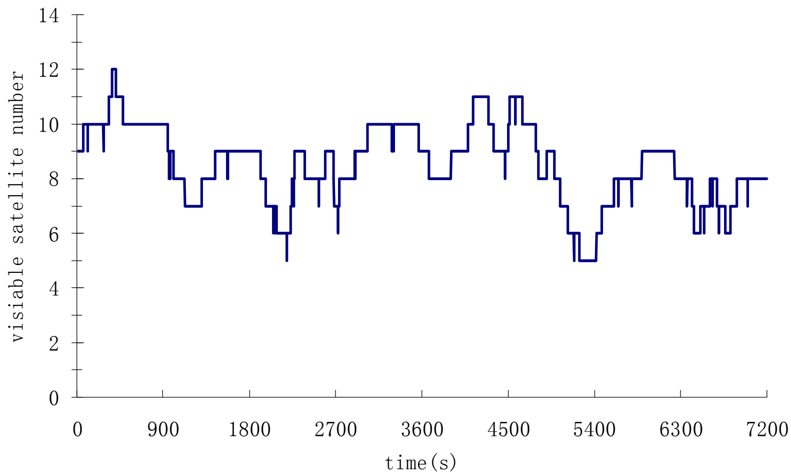


Figure 1. Visible GPS satellites number

The random errors within the carrier phase measurements were also simulated based on normal distributions with a standard deviation of 5mm. The outliers, about 20-30 standard deviation (0.3-0.5 cycle of carrier phase), were added to the different measurements randomly and make them greater or smaller.

When the carrier phase measurements are sufficiently available, a relative positioning solution of higher accuracy can be obtained by combination of the pseudorange

position and the carrier phase measurements, in which the pseudorange position results were taken as a datum of relative solution. The measurements taken on each receiver collected and aligned so that single differences were taken between measurements from the same GPS satellite. The integer ambiguity resolution methods exist can be applied here to fix the integers [6]. Double-differences and single differences were tried and both techniques demonstrated similar performance.
Figures 4 and Fig. 5 show the relative position results of LS estimator and robust estimator under no outlier situation.

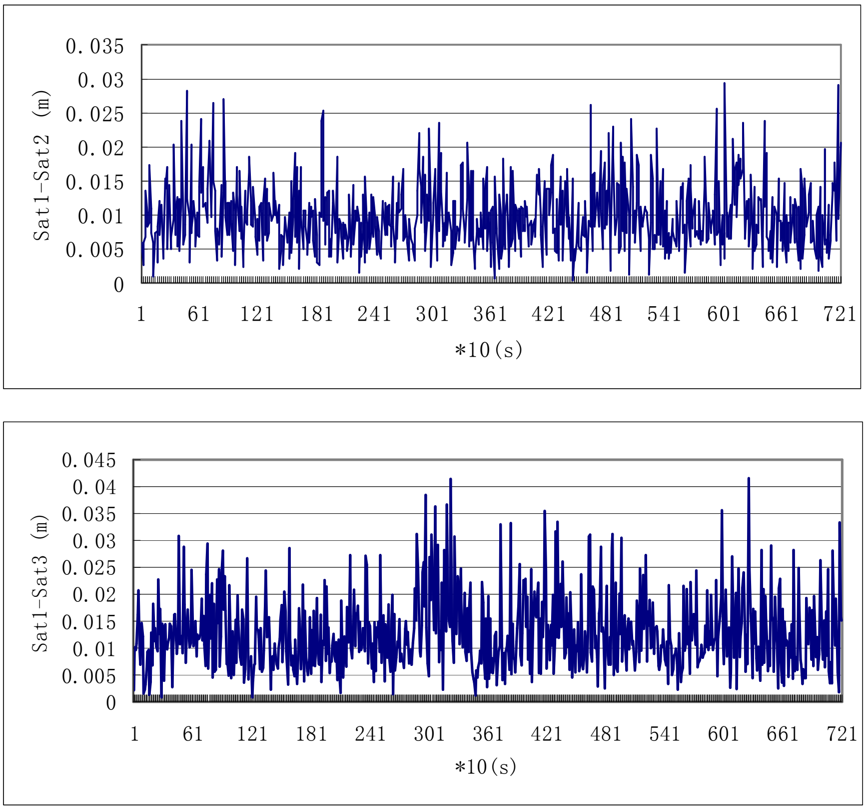
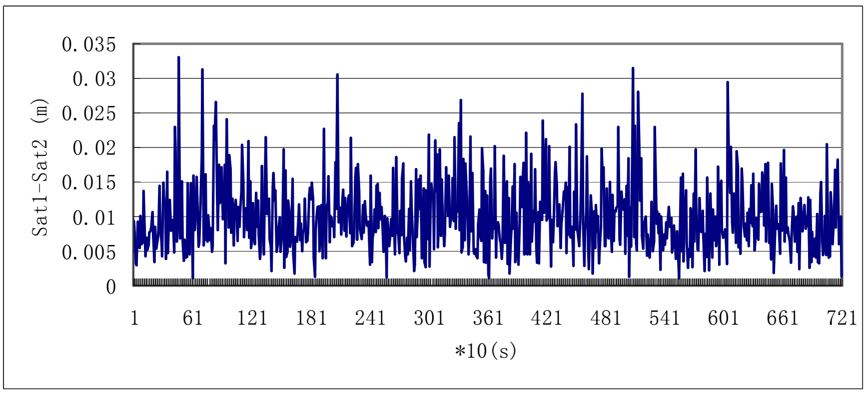


Figure 4. LS estimator under none outlier situation



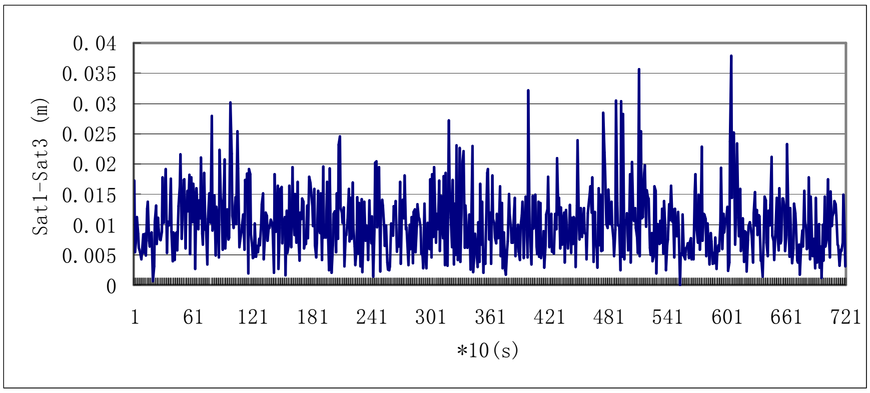
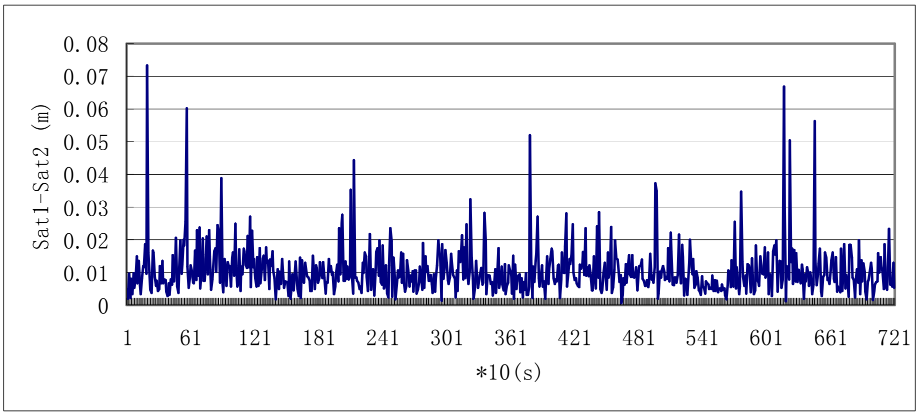


Figure 5. Robust estimator under no outlier situation

We may see that the least-square estimator and the IGG3 robust estimator are equivalent when there are no outliers. The results of relative position from two estimator are under 0.03m in most epoch.

Figures 6 and Fig. 7 show the relative position results of LS estimator and robust estimator under outlier situation.



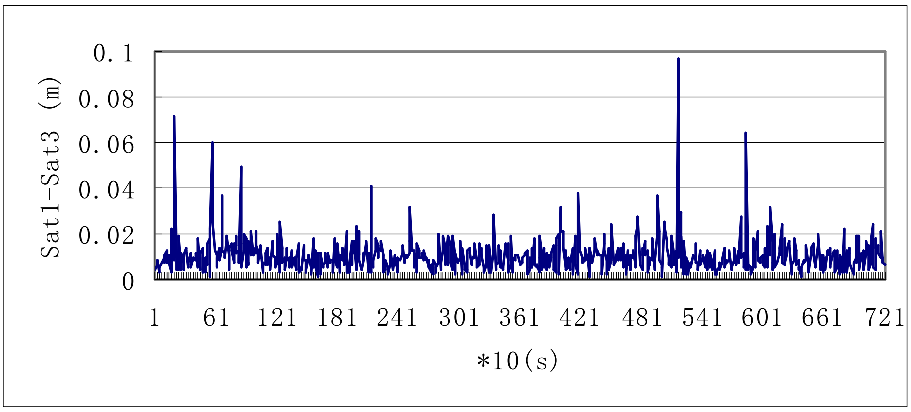
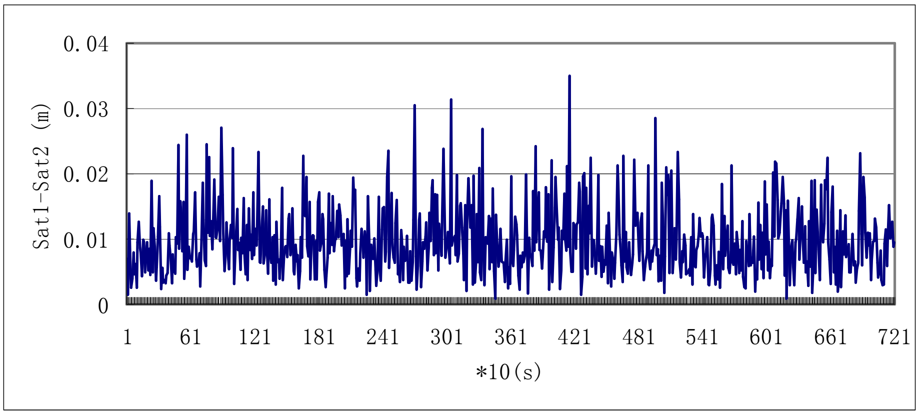


Figure 6. LS estimator under outlier situation



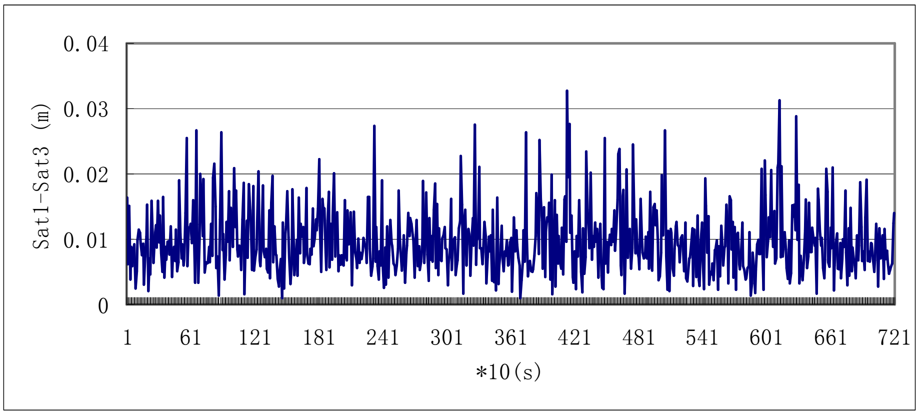


Figure 7. Robust estimator under outlier situation

It can be seen that the results of IGG3 robust estimator are different form least-square estimator The results show that the relative position of IGG3 robust estimator are almost under 0.03m and much better than least-square estimator on the average.

4. Conclusion

When spacecraft formation flying uses GNSS for its position, the receiver will suffer more faulty observations for its operation environment. These faulty observations will affect the position results. The robust estimator can be used to mitigate the influence of faults or outliers in this issue. The IGG3 scheme which is established based on an M estimation and the principle of equivalent weight is carry out for outlier situation. Comparing with least-square estimator, the quality control of IGG3 scheme is effective. Under situation of 0.3-0.5 cycle of carrier phase outlier in measurement, the IGG3 robust estimator keeps the accuracy almost under 0.03m which is the same in none outlier situation.

5. Acknowledgement

This work was sponsored by Natural Science Foundation of China (41274023).

6. References

[1] J. I. Park et al, "Hardware-in-the-loop simulations of GPS-based navigation and control for satellite formation flying," Advances in Space Research Vol. 46, 2010, pp. 1451-1465.

[2] Wang Wei, "Precise State Determination of Formation Satellite for Earth Observation Using GPS," Ph.D. dissertation, Information Engineering University, Zhengzhou, China, 2004.

[3] Psiaki. M. L., Mohiuddin. S., "Modeling, analysis, and simulation of GPS carrier phase for spacecraft relative navigation," Journal of Guidance Control and Dynamics, Vol. 30(6), 2007, pp.1628–1639
.

[4] Wang Wei, Xi Xiaoning. ATTITUDE AND RELATIVE POSITIONING ESTIMATION FOR FORMATION CONSTELLATION USING GPS. Chinese Journal of Space Science, 2002,22(2), pp. 163-168

[5] YANG Yuan-xi, "Robust Estimation for Dependent Observations," Manuscripta Geodaetica, Vol. 19, 1994, pp.10-17.

[6] S. Han, C. Rizos, "Comparing GPS Ambiguity Resolution Techniques," GPS World, Oct. 1997.

[7] YANG Yuan-xi, "The Theory and Application of Robust Estimation," Bayi Publishing House, Beijing, 1993.

[8] Yang Y , Song L，Xu T, "Robust Estimator for Correlated Observations Based on Bi-factor Equivalent Weights," Journal of Geodesy, Vol. 76(6), 2002, pp. 353—358